MODULATED IMPULSIVE FORCES
FOR HAPTIC RENDERING OF RIGID CONTACT

Benjamin Birch
Department of Mechanical Engineering
University of Victoria
Victoria, BC V8W 3P6 Canada
Email: benbf@uvic.ca

Daniela Constantinescu
Department of Mechanical Engineering
University of Victoria
Victoria, BC V8W 3P6 Canada
Email: danielac@me.uvic.ca

ABSTRACT
This paper proposes a method for the passive rendering of trains of impulsive forces to users when users contact hard virtual surfaces with a rigid virtual tool. The goal is to improve the sense of presence in virtual environments through enabling users to perceive various transient dynamics at contact onset. The proposed method computes the impulsive forces in the feedback loop based on Newton’s restitution hypothesis, and superimposes them on traditional penalty-based feedback. The method accounts for the energy transferred between the user and the virtual environment by the penalty forces via re-computing the impulsive forces throughout the duration of the transient dynamics. A preliminary user study shows that the modulated impulses are perceptually distinguishable from haptic rendering of collisions via a single impulse.

1 INTRODUCTION
Mimicking rigidity is not straightforward in haptics. A key challenge arises because haptic devices typically render contact stiffness through the stiffness of a position-based control loop. The gain of this feedback loop is limited by factors like the Z-width of the device, sampling frequency, time delays, sensor quantization, and noise [1]. The direct result of the finite stiffness of the control loop is that rigid virtual objects feel softer than real objects.

Yet, psycho-physical experiments have shown that contact stiffness in only one factor affecting the perceived rigidity of virtual environments [3]. For example, the apparent stiffness of virtual objects can be increased by applying large forces to users when contacts arise. This is because large forces increase rate hardness [3], which is another key factor affecting the perceived rigidity of virtual environments. Introduced in [3], rate hardness is defined as the rate of change of contact force over the relative approach velocity at the moment when contact is initiated. Existing research has increased rate hardness through applying impulsive forces to users. Computed either on-line [4] or off-line [6], the impulsive forces generate large rates of change of the contact forces when new contacts occur and reduce user's penetration into virtual walls.

The impulsive forces arise from contact models that represent collisions explicitly and thus, render rigid contact more realistically than the traditional penetration-based penalty forces. However, the impulsive forces are typically derived based on Newton's restitution hypothesis in haptics applications. Newton's hypothesis is an algebraic collision law. It predicts the velocity change of the contacting objects based on the coefficient of restitution \( e \). It does not model the dynamics of the interaction during collision. In turn, the impulsive forces derived based on Newton's law do not portray these dynamics to users.

Several studies [7]-[10] have investigated how the realism of rigid contact in virtual environments can be increased through super-imposing transient vibratory forces onto the typical penalty-based force feedback. These studies
have generally proposed pre-computing the super-imposed vibrations based on models of the haptic interface and/or of the user's hand. Collected data indicate that vibratory surfaces increase users' sense of presence in virtual environments regardless of the challenges involved in adjusting vibration parameters to closely replicate real contacts. Users prefer traditional penalty-based feedback only when the vibrating virtual surfaces feel "active".

The present work is motivated by the documented user preference for perceiving transient vibrations upon contact with rigid objects. It proposes a technique to modulate physically-based collision impulses computed in the feedback loop when users touch hard virtual surfaces. The modulated collision impulses are applied as transient impulsive forces and render the fast collision dynamics to users. A preliminary user study illustrates that users can distinguish the modulated impulsive forces from a single impulse.

The paper starts with the outline of the contact model that underlies the proposed haptic rendering method in Section 2. The virtual environment dynamics are briefly overviewed in Section 3. The modeling assumptions that underlie collision resolution and the proposed impulse modulation technique are discussed in Section 4. The preliminary user study is detailed in Section 5. The paper ends with conclusions and directions for future work.

2 CONTACT MODEL

The impulse modulation technique proposed herein uses a model of rigid contact that is based on the impulse-augmented penalty contact model introduced in [5]. In the impulse-augmented penalty contact model, a contact has infinite stiffness when it arises and finite stiffness afterwards, as schematically represented in Figs. 1(a) and 1(b). The switching stiffness enables the haptic simulation to distinguish collisions from resting contact and to compute passive interaction impulses. These passive impulses are then applied to users as impulsive forces during one simulation step (Fig. 1(c)). The impulsive forces increase rate hardness without requiring increased virtual contact stiffness and damping. However, they render collisions as instantaneous events and cannot be used to provide users with vibration feedback similar to that recorded when users tap on real hard objects [9].

The present work proposes to amend the impulse-augmented penalty contact model in [5] to permit a richer representation of the collision dynamics. The refinement is achieved via conceiving that collision rendering lasts several steps of the haptic simulation. The finite duration of collision rendering introduces additional degrees of freedom in the design of the haptic rendering algorithm and allows various transient dynamics to be portrayed to users. Specifically, it allows trains of impulsive forces of varying frequency and magnitude to be applied to users at contact onset. An example of contact forces that can be used to represent collisions to users over multiple simulation steps is illustrated in Fig. 2.

3 VIRTUAL ENVIRONMENT DYNAMICS

In the impulse-augmented penalty contact model with finite duration collision rendering, rigidity must be enforced: (i) via constraint-based collision resolution techniques at the moments when collision impulses arise during colliding contact; and (ii) via penalty-based forces during resting contact. This is done as in [5] and only briefly overviewed herein.

Consider a system of \( b \) rigid bodies interacting through \( c \) contacts modeled as described in Section 2, collectively called a contact group. In the contact group, bodies are numbered such that body 1 is the virtual tool held by the user. The second order dynamics of the contact group can be written as:

\[
Ma = \begin{bmatrix} F_{\text{user}} \\ 0 \end{bmatrix} + G - B + J^T F
\]
In Eqn. (1), bold symbols represent vectors and matrices; \( \mathbf{M} \) is a block diagonal matrix having the body mass matrices \( \mathbf{M}_i \) of all bodies \( i = 1, \ldots, b \) in the group on its diagonal; \( \mathbf{a} = \left( \mathbf{v}_i \right) \left( \omega_i^T \right) \cdots \left( \mathbf{v}_s \right) \left( \omega_s^T \right) \) is the acceleration of the contact group, and is obtained by concatenating the linear \( \mathbf{v}_i \) and angular \( \omega_i \) accelerations of all bodies \( i = 1, b \) in the group; \( \mathbf{F}_{\text{app}} \) is the wrench (i.e., force and torque) applied by the user at the centre of mass of the virtual tool;

\[
\mathbf{G} = \begin{pmatrix} 
\mathbf{G}_1 & 0 & \cdots & \mathbf{G}_s & 0 
\end{pmatrix}^T
\]

are gravity forces;

\( \mathbf{B} = (\mathbf{B}_i \cdots \mathbf{B}_s)^T \) represent Coriolis and centripetal effects;

\( \mathbf{F} = (\mathbf{F}_i \cdots \mathbf{F}_s)^T \) are contact forces; and \( \mathbf{J}_j^T = [\pm \mathbf{J}_j^T] \) is the contact Jacobian. Furthermore, \( \mathbf{J}_j^T \) is the Jacobian of the \( j \)-th contact with respect to the centre of mass of body \( i \), computed via \( \mathbf{J}_j = [\mathbf{1}_{3s} - (\mathbf{r}_j \times)] \) if body \( i \) is one of the bodies involved in the \( j \)-th contact, and via \( \mathbf{J}_j = \mathbf{0}_{3s} \) otherwise, \( \mathbf{r}_j \) is the position of the \( j \)-th contact with respect to the centre of mass of body \( i \), and \( (\mathbf{r}_j \times) \) is the cross product operator. The sign of \( \mathbf{J}_j \) is such that the corresponding contact forces obey the law of action and reaction: if the \( j \)-th contact is between bodies \( i \) and \( l \) and the sign of \( \mathbf{J}_j \) is positive, then the sign of \( \mathbf{J}_l \) is negative.

When a new contact occurs in the contact model of [5], the contact group enters the collision state. The velocity-impulse dynamic equations of the group are used to resolve the collision:

\[
\mathbf{M} \dot{\mathbf{v}} = \mathbf{M} \dot{\mathbf{v}}_0 + \int_{t_0}^t \mathbf{F}_c \, dt = \mathbf{M} \dot{\mathbf{v}}_0 + \mathbf{J}_j^T \mathbf{p},
\]

where \( \mathbf{v} \) is the velocity of the contact group after collisions, \( \mathbf{v}_0 \) is the contact group velocity before collision, and \( \mathbf{p} = \int_{t_0}^t \mathbf{F} \, dt \) is the contact impulse. Its computation is presented in Section 4. In Eqn. (2), the impulses due to the user, gravity, and centripetal forces are neglected, because the duration of collisions is considered small enough to allow collisions to be treated as instantaneous events (i.e., \( t \to t_0 \)). To preserve the instantaneous nature of collisions, the impulse modulation method proposed herein selects the duration of collision rendering smaller than users' reaction time\(^2\) and postulates that collisions can be rendered to users as multiple impulsive forces during this interval.

### 4 IMpulSe MODulation

#### 4.1 Assumptions

Two types of assumptions are used to compute the contact impulse \( \mathbf{p} \) in Eqn. (2): (I) assumptions related to collision resolution and to the physics of the interaction; and (II) assumptions related to collision rendering and to the design parameters introduced by the hypothesis that collisions last multiple simulation steps.

I. The assumptions related to collision resolution employed in this work are:

a. frictionless contacts:

\[
\mathbf{J}_j^T = [\mathbf{J}_j^T \mathbf{n}].
\]

In Eqn. (3), \( \mathbf{n} \) is the normal to the \( j \)-th contact.

b. Newton’s restitution hypothesis:

\[
\nu = -e \nu_0
\]

In Eqn. (4), \( e \in [0,1] \) is the coefficient of restitution, \( \nu \) is the relative velocity of the contacting bodies along the contact normal, and the index zero represents pre-collision quantities.

As discussed in [5], these assumptions allow the computation of a passive contact impulse at contact onset, according to:

\[
\mathbf{p} = -(1 + e)(\mathbf{J}_j \mathbf{M}^{-1} \mathbf{J}_j^T) \mathbf{J}_j \nu_0.
\]

In Eqn. (5), \( (\mathbf{J}_j \mathbf{M}^{-1} \mathbf{J}_j^T) \) represents pseudo-inversion and accounts for simultaneous redundant collisions.

II. The assumptions related to collision rendering set the values of the design parameters employed in portraying the dynamics of collisions to users. They include:

a. the duration of colliding contact, \( t_c \). This is chosen smaller than user’s reaction time, i.e., no larger than 100msec [6].

b. the frequency of the train of impulses, \( f \). The time step of the simulation \( h \) sets an upper bound on the choice of \( f \). Conversely, the duration of colliding contact \( t_c \) sets a lower bound on \( f \). Within these bounds, \( f \) could be chosen to match the frequency content of various real contacts, for example as identified in [10]. However, whether such a choice allows users to match a virtual contact to the real contact it mimics is a question that requires careful investigation via user studies.

---

\(^2\) Given that human action band is approximately 10Hz [6], the duration of collision rendering is chosen to be less than 100 msec.
Given \( t \) and \( f \), other parameters of the haptic rendering algorithm can be computed for a given time step of the virtual environment simulation \( h \):

- the number of steps during which the transient collision dynamics are rendered to users, \( n \):
  \[
  n = \frac{t}{h} \tag{6}
  \]

- the number of time steps between consecutive impulses, \( n_i \):
  \[
  n_i = \frac{1}{f \cdot h} \tag{7}
  \]

The design of the proposed impulse modulation scheme is guided by user preferences identified in prior psycho-physical studies [6]. In particular, users dislike bouncy/active contacts, and prefer a sudden stop and high frequency vibrations upon contact. This work avoids bouncy/active contacts via applying trains of passive impulsive forces to users. It rapidly stops users’ motion into the touched objects through rendering perfectly plastic collisions, i.e., \( e = 0 \). Lastly, it implements trains of impulses with frequency as high as permitted by the time step of the simulation. The modulation scheme developed according to these design criteria is introduced in the following section.

4.2 Train of Impulsive Forces Superimposed on Penalty Forces

As mentioned earlier, impulse modulation is based on the assumption that the transient collision dynamics can be represented to users via a train of impulsive forces rendered over a limited time interval at contact onset. This assumption is reasonable if the finite duration during which the impulsive feedback is applied is smaller than the frequency of human voluntary motions. In this case, users still perceive collisions as instantaneous events, because they can respond only to the cumulative effect of all impulses in the train rather than to each impulsive force individually. A schematic representation of the proposed collision rendering algorithm is shown in Fig. 3. In this figure, \( \delta t \) represents the time interval measured from the contact onset.

The \( i \)-th impulse is derived via:

\[
\mathbf{p}_i = -(1 + 0) (\mathbf{J}^T \mathbf{M}^{-1} \mathbf{J}^T) \mathbf{J} \mathbf{v}_{i,0} \cdot \frac{n}{n_i} - i + 1
\]

\[
\mathbf{F}_i = \frac{\mathbf{J}^T \mathbf{p}_i}{h} = -\frac{J^T (J^T M^{-1} J^T) J \mathbf{v}_{i,0}}{h \left( \frac{n}{n_i} - i + 1 \right)} \tag{9}
\]

Each modulated impulsive force decreases the kinetic energy of the virtual tool at the moment when it is applied, while the user-applied wrench increases its kinetic energy between consecutive impulses. Hence, the velocity of the
virtual tool may decrease or increase between consecutive impulses. Therefore, impulse modulation is suitable only for perfectly plastic collisions, \( e = 0 \), for which the post-collision velocity \( v = \mathbf{0} \) is compatible with each intermediate pre-collision velocity \( v_{i,o} \).

Figure 3 shows that the impulsive forces are superimposed on penalty feedback. This is because, depending on the initial conditions at contact onset (user-applied force, pre-collision velocity, duration of collision rendering, and frequency of the impulsive forces), modulated impulsive feedback alone may result in large constraint penetration at the end of collision rendering. The large penetration would result in large penalty forces that might destabilize the interaction. This difficulty is avoided by superimposing the impulsive feedback on traditional penalty feedback.

Lastly, the energy transferred to the user by the penalty forces is implicitly accounted for in Eqn. (8). Specifically, this energy is represented in \( v_{i,o} \), and is thus included in the energy to be dissipated via impulsive feedback.

5 USER STUDY IMPLEMENTATION AND RESULTS

A preliminary user study has been performed to determine users’ ability to discern: (a) the modulated impulsive feedback from a single impulsive force; and (b) high frequency from low frequency modulated impulsive forces. The study involved 9 volunteer students from the University of Victoria. Seven of the 9 volunteers were engineering students, one was female, and no volunteer had any previous experience with haptic interfaces. A total of three wall types were presented to the volunteer users. Table 1 details the three wall types and their defining properties.

Table 1. TYPES OF WALLS USED IN THE USER STUDY

<table>
<thead>
<tr>
<th>Wall type</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>N/A</td>
</tr>
<tr>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
</tbody>
</table>

The users were allowed to familiarize themselves with the three walls and they were informed of the mathematical differences between the walls. The users were then put though two tests, each of which consisted of differentiating between two wall types. Specifically, users were asked to differentiate between wall 0 (single impulse) and wall 1 (high frequency \( f = 250\text{Hz} \) modulated impulses) in the first test. They were asked to differentiate between wall 1 (high frequency modulated impulses) and wall 2 (low frequency \( f = 50\text{Hz} \) modulated impulses). The test required users to tap on the first wall three times and then on the second wall three times followed by 20 series of three taps that would be either of the two wall types in random order. Users were asked to identify the wall type after each series of three taps. Users’ view of the virtual environment was blocked during the tests and white noise was played through headphones to block any auditory cues. Table 2 presents the results of the preliminary user study.

Table 2. RESULTS OF THE USER STUDY

<table>
<thead>
<tr>
<th>Test #</th>
<th>Property tested</th>
<th>% correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wall 0 vs. wall 1</td>
<td>82.22%</td>
</tr>
<tr>
<td>2</td>
<td>Wall 1 vs. wall 2</td>
<td>63.89%</td>
</tr>
</tbody>
</table>

As seen from Tab. 2, users distinguished the modulated impulses from the single impulse better than they distinguished the high frequency from the low frequency modulated impulses. The difference between the single impulse (wall 0) and the train of impulses (wall 1) was noticed by most of the users. Two thirds of the users had 90% or higher accuracy in differentiating between the two wall types but one third of the users had difficulty distinguishing between them (40%-65%).

The test to determine the difference between the trains of impulses rendered at frequencies of 250 Hz (wall 1) and 50Hz (wall 2) produced mixed results. Approximately 44% of the users had success (90% or greater accuracy) while one third of the users had no success (30% to 45% accurate). One user appeared to be able to tell the difference between the two types but could not discern which was which, only that they were different. Therefore when they began and guesses the fist wall type incorrectly they then proceeded to be incorrect for most of the remainder of the test. This user had 10% success at determining which wall was which. However, this result shows that they could differentiate between the two walls. To some extent, this was a recurring problem for other users. They expressed that, as the test progressed, they could tell the walls apart but had forgotten which feeling was associated with what wall. This result shows that users may be able to discriminate among various virtual contacts better than they are able to classify them. Given that users would not be deprived of visual feedback in typical haptic applications, the ability to discriminate the feel of a variety of contacts may be enough to improve presence in haptically-enabled virtual environments.

6 CONCLUSION

The purpose of this research was to develop a passive impulse modulation method for rendering various transient dynamics to users upon rigid contact. The passivity of the method has been guaranteed via accounting for the energy transferred between the user and the virtual environment
through penalty-based feedback. Compared to pre-computed transient vibrations, the modulation scheme described in this work is guaranteed passive. Compared to collision rendering via impulsive and penalty forces as proposed in [5], it provides users with additional haptic cues at the price of rendering only perfectly plastic contacts. The perceptual relevance of the proposed impulse modulation technique has been investigated via a preliminary user study. This study has shown that users can distinguish a train of impulses from a single impulse. Users can also differentiate between trains of high and low frequency impulses, although they appear to have difficulty remembering which haptic cues correspond to which impulse frequency.

The results of the initial user study support the original hypothesis that trains of passive impulsive forces can render richer transient dynamics to users at contact onset than a single impulsive force. Upcoming work will investigate; (i) how these richer transient dynamics can be used to enable users to distinguish between contact with various hard surfaces (e.g., wood on wood or metal on metal); (ii) alternative passive methods for rendering the transient dynamics of rigid contact to users.

ACKNOWLEDGEMENT

The authors would like to thank Professor Tim Salcudean for providing the planar haptic interaction system used for implementing and validating the proposed impulse modulation technique.

REFERENCES