**Constant Acceleration (Parabolic Motion)**

This scheme produces the smallest possible values for acceleration. The displacement diagram is divided into two halves: constant acceleration and constant deceleration. The shape of each half are mirror-image parabolas. The abrupt change of accelerations at the beginning, at the transition point, and at the ending of the motion cause undesirable vibrations (infinite jerk). The transition point occurs at \( \Theta^* = \beta / 2 \).

\[ \begin{align*}
\text{Constant Acceleration Rise} & \\
\text{Follower Displacement: } S^* &= L \left( \frac{\Theta^*}{\beta} \right)^2 \\
\text{Follower Velocity: } \dot{S}^* &= \frac{4L \Theta^*}{\beta^2} \Theta^* \\
\text{Follower Acceleration: } \ddot{S}^* &= \frac{4L}{\beta^2} \left( \Theta^* - \frac{\dot{\Theta}^*}{\beta^2} \Theta^* \right) \\
\text{Constant Acceleration Fall} & \\
\text{Follower Displacement: } S^* &= L \left[ 1 - 2 \left( \frac{\Theta^*}{\beta} \right)^2 \right] \\
\text{Follower Velocity: } \dot{S}^* &= -\frac{4L \dot{\Theta}^*}{\beta^2} \Theta^* \\
\text{Follower Acceleration: } \ddot{S}^* &= 4L \left( -\frac{\Theta^*}{\beta} + \frac{\dot{\Theta}^*}{\beta^2} \Theta^* \right) \\
\end{align*} \]
Simple Harmonic Motion

This motion scheme incorporates a portion of a sine wave. This scheme shows a notable improvement over the other schemes, as shown on the smoothness of the motion curves. Acceleration has finite values throughout the follower motion. Nevertheless, at the beginning and end of harmonic motion, there is a step change of acceleration yielding infinite jerk.

\[ S^* = \frac{L}{2} \left( 1 - \cos \frac{\pi \theta^*}{\beta} \right) \]

\[ S^* = \frac{L}{2} \left( 1 + \cos \frac{\pi \theta^*}{\beta} \right) \quad (0 \leq \theta^* \leq \rho) \]

\[ V^* = \frac{L}{2} \left( \frac{\pi \theta^*}{\beta} \right) \sin \left( \frac{\pi \theta^*}{\beta} \right) \]

\[ V^* = -\frac{L}{2} \left( \frac{\pi \theta^*}{\beta} \right) \sin \left( \frac{\pi \theta^*}{\beta} \right) \quad (0 \leq \theta^* \leq \rho) \]

\[ a^* = \frac{L}{2} \left( \frac{\pi \theta^*}{\beta} \right)^2 \cos \left( \frac{\pi \theta^*}{\beta} \right) \]

\[ a^* = -\frac{L}{2} \left( \frac{\pi \theta^*}{\beta} \right)^2 \cos \left( \frac{\pi \theta^*}{\beta} \right) \quad (0 \leq \theta^* \leq \rho) \]
Cycloidal Motion

Cycloidal Motion is another motion scheme derived from trigonometric functions. This scheme provides very smooth motion curves. The acceleration values at the start and finish are zero. As a result, jerk remains finite throughout the entire motion. Thus, this scheme can be used for high-speed applications.

\[
S^* = L \left( \frac{\Theta^*}{\beta} - \frac{1}{2\pi} \sin \left( \frac{2\pi \Theta^*}{\beta} \right) \right) \quad (0 \leq \Theta^* \leq \beta)
\]

\[
V^* = \dot{S}^* = \frac{L\dot{\Theta^*}}{\beta} \left( 1 - \cos \left( \frac{2\pi \Theta^*}{\beta} \right) \right) \quad (0 \leq \Theta^* \leq \beta)
\]

\[
A^* = \ddot{S}^* = \frac{2\pi L (\dot{\Theta^*})^2}{\beta} \sin \left( \frac{2\pi \Theta^*}{\beta} \right) \quad (0 \leq \Theta^* \leq \beta)
\]