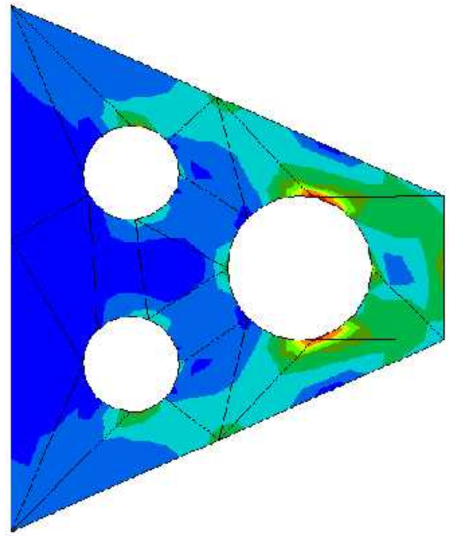
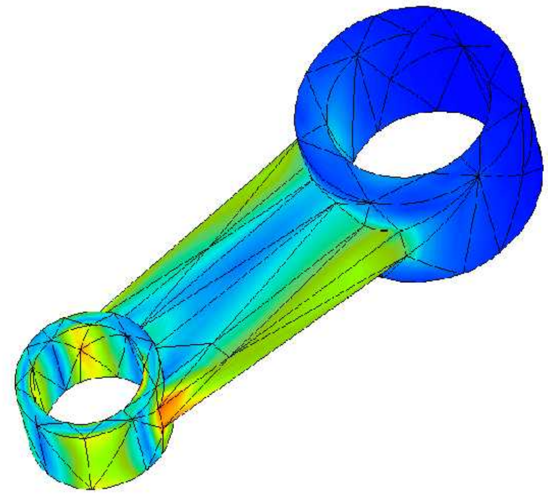
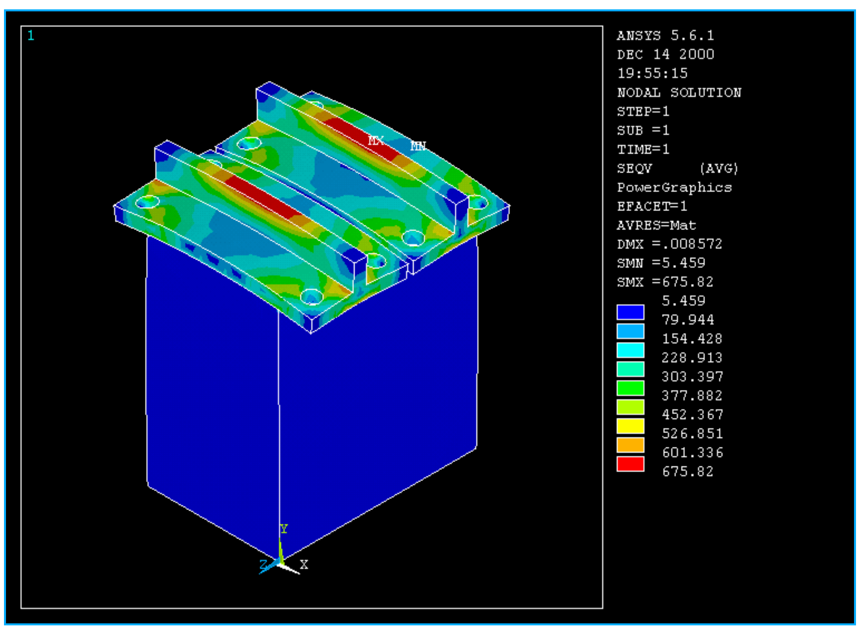
Introduction to Design Optimization

Various Design Objectives

Minimum Weight
(under Allowable Stress)

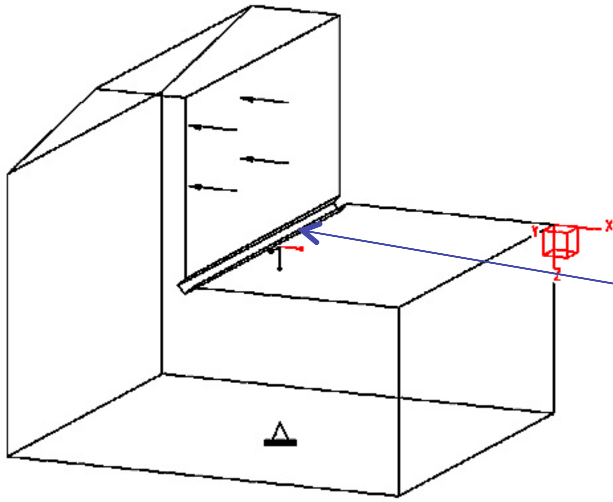






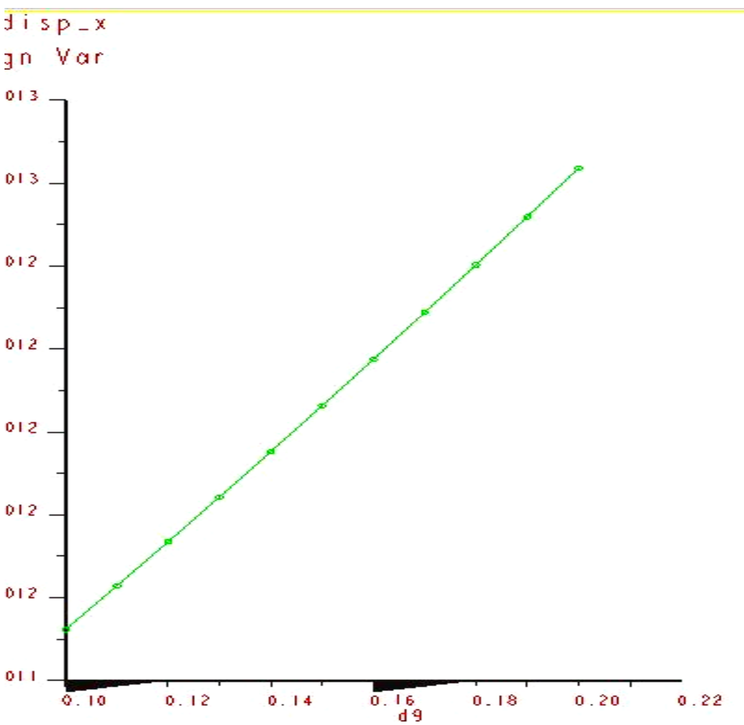
A PEM Fuel Cell Stack with Even Compression over Active Area

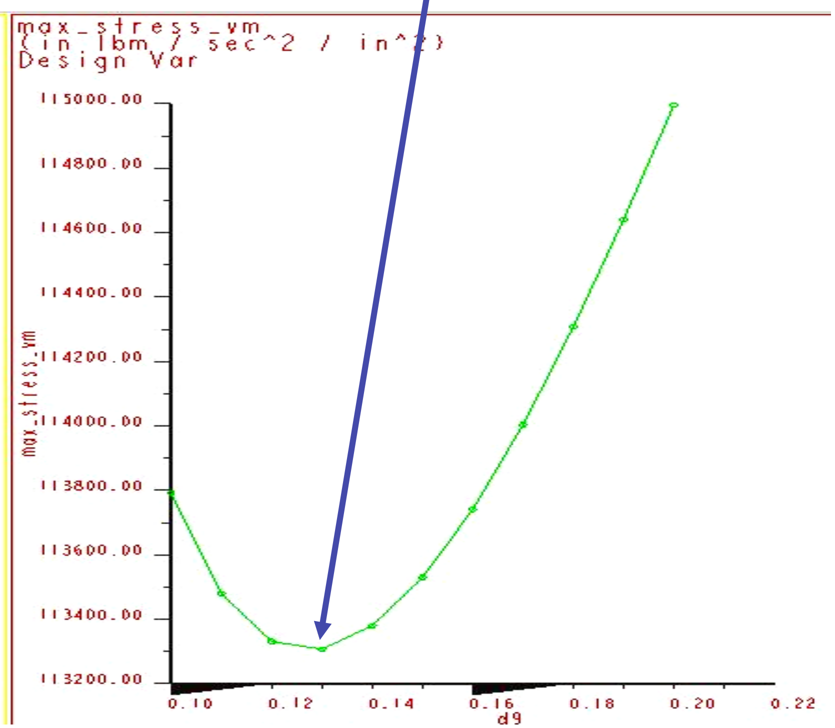
(Minimum Stress Difference)



Minimum Maximum Stress in the Structure

Optimized Groove Dimension
to Avoid Stress Concentration
or Weakening of the Structure





Engineering Applications of Optimization

- **Design** - determining design parameters that lead to the best "performance" of a mechanical structure, device, or system.

  "Core of engineering design, or the systematic approach to design" (Arora, 89)

- **Planning**
  - production planning - minimizing manufacturing costs
  - management of financial resources - obtaining maximum profits
  - task planning (robot, traffic flow) - achieving best performances
- **Control and Manufacturing** - identifying the optimal control parameters for the best performance (machining, trajectory, etc.)
- **Mathematical Modeling** - curve and surface fitting of given data with minimum error

Commonly used tool: OPT function in FEA; MATLAB Optimization Toolbox

What are common aspects in optimization problems?

- There are multiple solutions to the problem; and the optimal solution is to be identified.
- There exist one or more objectives to accomplish and a measure of how well these objectives are accomplished (measurable performance).
- Constraints of different forms (hard, soft) are imposed.
- There are several key influencing variables. The change of their values will influence (either improve or worsen) the "measurable performance" and the degree of violation of the "constraints."

Solution Methods

- Optimization can provide either
  - a closed-form solution, or
  - a numerical solution.
- Numerical optimization systematically and efficiently adjusts the influencing variables to find the solution that has the best performance, satisfying given constraints.
- Frequently, the design objective, or cost function cannot be expressed in the form of simple algebra. Computer programs have to be used to carryout the evaluation on the design objective or costs. For a given design variable, α, the value of the objective function, $f(\alpha)$, can only be obtained using a numerical routine. In these cases, optimization can only be carried out numerically.

α → **Computer Program (no simple algebra)** → $f(\alpha)$

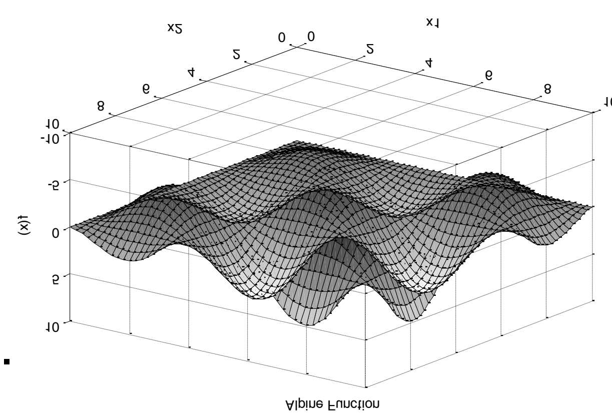
e.g. Minimize the maximum stress in a tents/tension structures using FEA.

Definition of Design Optimization

An optimization problem is a problem in which certain parameters (*design variables*) needed to be determined to achieve the best measurable performance (*objective function*) under given *constraints*.

Classification of the Optimization Problems

- **Type of design variables**
  - optimization of continuous variables
  - integer programming (discrete variables)
  - mixed variables
- **Relations among design variables**
  - nonlinear programming $\quad e.g.\ \ f(X) = Ae^{-x_1} + Bx_2$
  - linear programming $\quad e.g.\ \ f(X) = c_1x_1 + c_2x_2 + \ldots + c_nx_n$
- **Type of optimization problems**
  - unconstrained optimization
  - constrained optimization
- **Capability of the search algorithm**
  - search for a local minimum
  - global optimization; multiple objectives; etc.



Automation and Integration

- Formulation of the optimization problems
  - specifying design objective(s)
  - specifying design constraints
  - identifying design variables
- Solution of the optimization problems
  - selecting appropriate search algorithm
  - determining start point, step size, stopping criteria
  - interpreting/verifying optimization results
- Integration with mechanical design and analysis
  - **black box** analysis functions serve as objective and constraint functions (e.g. FEA, CFD models)
  - incorporating optimization results into design

An Example Optimization Problem

Design of a thin wall tray with minimal material:

The tray has a specific volume, V, and a given height, H. The design problem is to select the length, l, and width, w, of the tray.

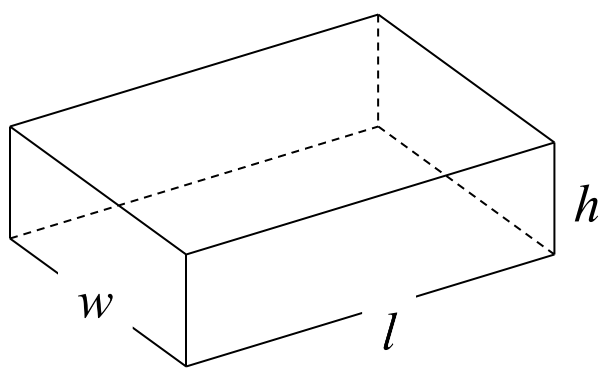
Given

$$lwh = V \qquad h = H$$

A “workable design”:

$$lw = \frac{V}{H}$$

Pick either l or w and solve for others



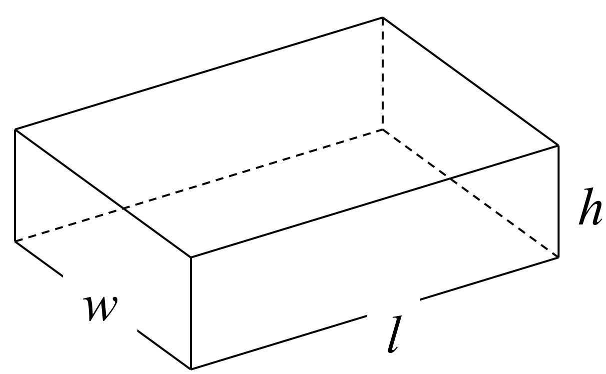
An "Optimal Design"

- The design is to minimize material volume, (or weight), where "T" is an acceptable small value for wall thickness.

Minimize $$V_m(w,l,h) = T(\underbrace{wl}_{bottom} + \underbrace{2lh + 2wh}_{sides})$$

Objective function

subject to
$$\left.\begin{aligned} lwh &= V \\ h &= H \\ l &\geq 0 \\ w &\geq 0 \end{aligned}\right\}$$
constraints (functions)



Design variables: w, l, and h.

Standard Mathematical Form

$$\min_{w.r.t. l,w,h} \quad T(wl + 2lh + 2wh)$$ - objective function

Subject to

$$\left.\begin{aligned} lwh - V &= 0 \\ h - H &= 0 \end{aligned}\right\}$$ - equality constraints

$$\left.\begin{aligned} -l &\le 0 \\ -w &\le 0 \end{aligned}\right\}$$ - inequality constrains

- variable bounds

$$\vec{x} = [l, w, h]^T$$ - design vector

- for use of any available optimization routines

Analytical (Closed Form) Solution

- Eliminate the equality constrains, convert the original problem into a single variable problem, then solve it.

from $h = H \;\&\; l\,w\,H = V;$ solve for l: $l = \dfrac{V}{Hw}$

thus

$$\min_{w} T\left(\frac{V}{Hw}w + 2\frac{V}{Hw}H + 2wH\right) \longrightarrow \min_{w} T\left(\frac{V}{H} + 2\frac{V}{w} + 2wH\right) = f(w)$$

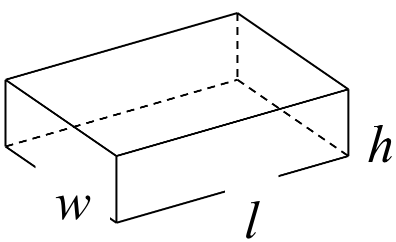
from $\dfrac{df(w)}{dw} = 0,\; we\,have\;\; w^2 = \dfrac{V}{H},\; then\; the\, design\; optimum\; w^* = \sqrt{\dfrac{V}{H}}$

- a stationary point

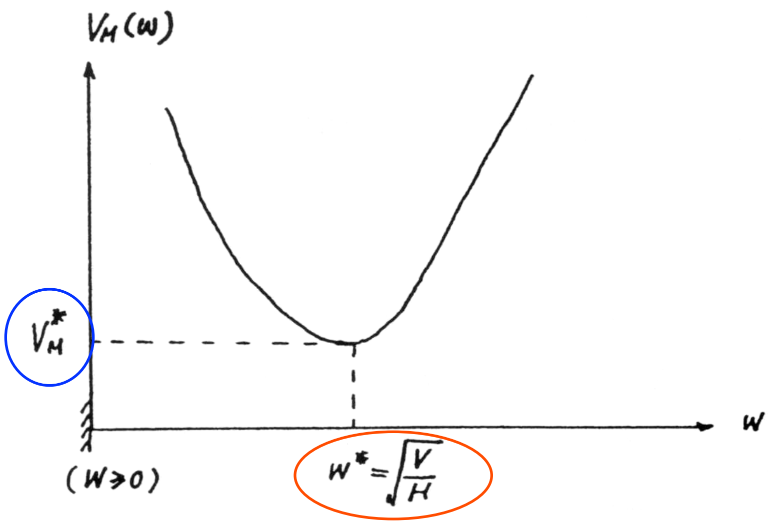
- Discard the negative value, since the inequality constraint is violated.
- The optimal value for l:

$$l^* = \frac{V}{Hw^*} = \sqrt{\frac{V}{H}} = w^*$$

$$V_M^* = T\left(\frac{V}{H} + 2hw^* + \frac{2V}{w^*}\right) = T\left(\frac{V}{H} + 4\sqrt{VH}\right)$$



Graphical Solution

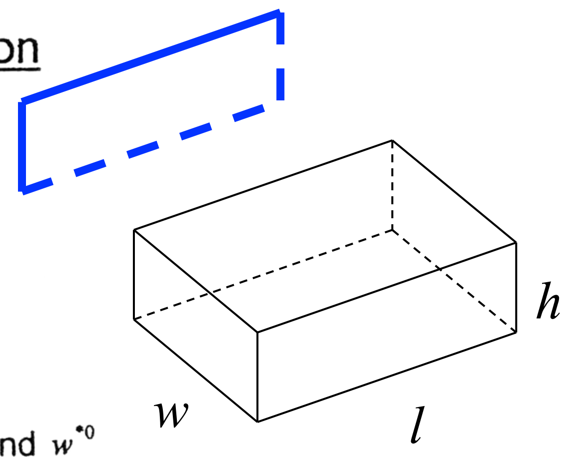


$w \leq \infty$ or $l \geq 0$

no width & length limitations
no violated constraints.

Change of Constraints and Their Influence to the Final Solution

Consider a modified problem:



$$\min_{l,w,h} \quad V_m = T\,(w \times l + 2 \times l \times h + 2 \times w \times h)$$

s.t.

$lwh = V$

$h = H$

$w \geq 0$

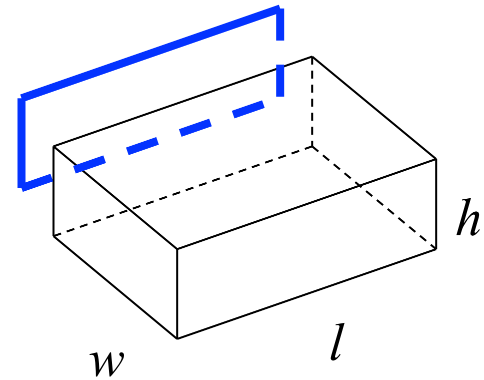
$l \geq 0$

Handled as an unconstrained problem and found w^{*0}

$\boxed{w \leq W \quad \text{maximum width / add a new constraint}}$

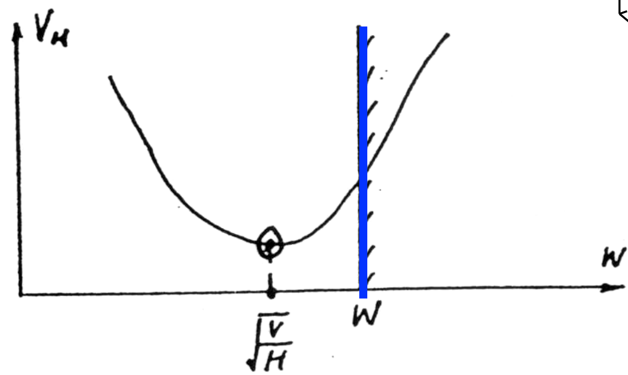
Follow the previous example:

unconstrained optimum: $w^{*0} = \sqrt{\frac{V}{H}}$

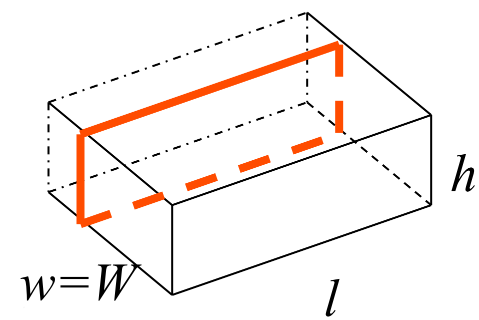


- For $W \geq w^{*0} = \sqrt{\frac{V}{H}}$

  $w^* = l^* = \sqrt{\frac{V}{H}}$



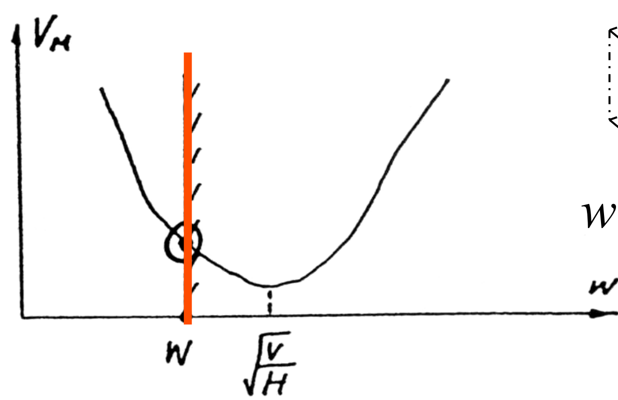
The constrained optimum is not changed, no active constraints.



- For $W \leq w^{*0} = \sqrt{\frac{V}{H}}$

  $w^* = W \neq w^{*0}$

  $l^* = \frac{V}{HW}$



Constraint $w \leq W$ is "active."

Procedures for Solving an Eng. Optimization Problem

- **Formulation of the Optimization Problem**
  - Simplifying the physical problem
  - identifying the major factor(s) that determine the performance or outcome of the physical system, such as costs, weight, power output, etc. – objective
  - Finding the primary parameters that determine the above major factors - the design variables
  - Modeling the relations between design variables and the identified major factor - objective function
  - Identifying any constraints imposed on the design variables and modeling their relationship – constraint functions
- **Selecting the most suitable optimization technique or algorithm to solve the formulated optimization problem.**

  - requiring an in-depth know-how of various optimization techniques.
- **Determining search control parameters**

  - determining the initial points, step size, and stopping criteria of the numerical optimization
- **Analyzing, interpreting, and validating the calculated results**

  An optimization program <u>does not guarantee</u> a correct answer, one needs to
  - prove the result mathematically.
  - <u>verify the result using check points</u>.

Standard Form for Using Software Tools for Optimization (e.g. MatLab Optimization Tool Box)

Denoting the optimization variables X, as a n-dimensional vector, where the n variables are its componets, and the objective function $F(X)$ we search for :

$$\mathbf{X}^* \in R^n \text{ so that } F(\mathbf{X}^*) = \min F(\mathbf{X})$$

subject to

$$\mathbf{X}_l \leq \mathbf{X}^* \leq \mathbf{X}_u$$

Regional constraints

$$G_i(\mathbf{X}^*) \geq 0 \qquad i = 1, 2, \ldots, m$$

and

Behavior constraints

$$H_j(\mathbf{X}^*) = 0 \qquad j = 1, 2, \ldots, q$$

Where m are the number of inequality constraints and q the number of equality constraints

Geometric Interpretation of the Objective Function

- The Objective function can be interpreted to be a surface of dimension *n* embedded in a space of dimension *n+1*. This is easy to visualize for a 2 parameter problem.

- The optimization process can be compared to “mountain climbing in a dense fog, having as only tool an altimeter”.