Hermite Curves Example

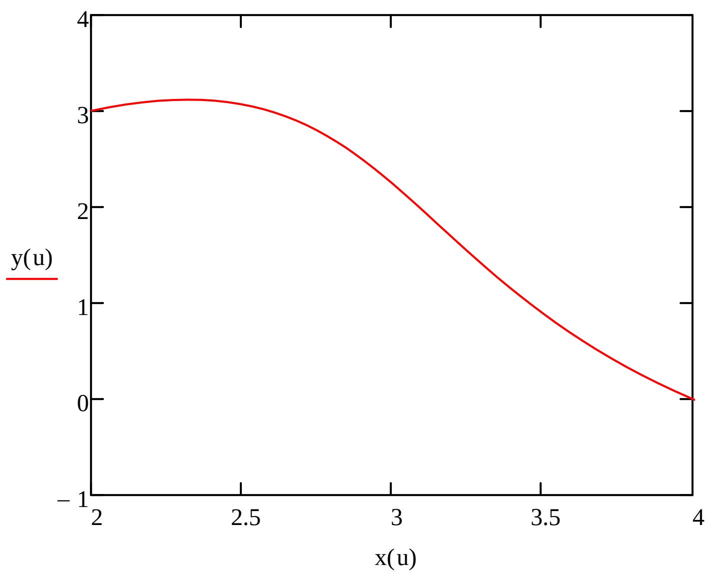
Curve
evaluation:

given $P_1(2,3,0)$, P_2 (4, 0, 0)
and derivatives P_1' (3,2) P_2' (3, -4) at the points P_1 and P_2

$$P(u) := \begin{pmatrix} 1 - 3\cdot u^2 + 2u^3 & 3u^2 - 2u^3 & u - 2u^2 + u^3 & -u^2 + u^3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 & 0 \\ 4 & 0 & 0 \\ 3 & 2 & 0 \\ 3 & -4 & 0 \end{pmatrix}$$

$x(u) := P(u)_{0,0}$ x values as function of u

$y(u) := P(u)_{0,1}$ y values as function of u



Bezier Curves Example

Control
points:

$P_0 := (1\ \ 1\ \ 0\ \ P_1 := (1.1\ \ 3\ \ 0)\quad P_2 := (1.5\ \ 2.5\ \ 0)\quad P_3 := (1.8\ \ 1\ \ 0)\quad P_4 := (2\ \ 1\ \ 0)$

$n := 4$ number of points

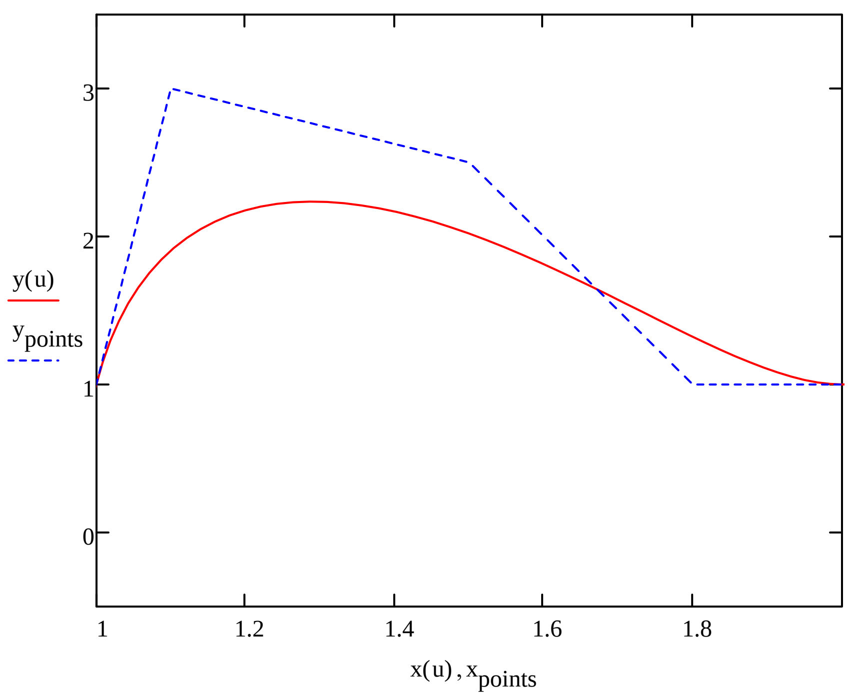
$P_b(u) := \sum_{i=0}^{n} \left[\frac{n!}{i!\,(n-i)!} u^i (1-u)^{n-i} \cdot P_i\right]$ curve evaluation

$x(u) := P_b(u)_{0,0}$ x values as function of u

$y(u) := P_b(u)_{0,1}$ y values as function of u

$x_{points} := \left[(P_0)_{0,0}\ \ (P_1)_{0,0}\ \ (P_2)_{0,0}\ \ (P_3)_{0,0}\ \ (P_4)_{0,0}\right]^T$

$y_{points} := \left[(P_0)_{0,1}\ \ (P_1)_{0,1}\ \ (P_2)_{0,1}\ \ (P_3)_{0,1}\ \ (P_4)_{0,1}\right]^T$



Bezier Example

$x_0 := 1$ $\quad x_1 := 1.5$ $\quad x_2 := 2.5$

$y_0 := 0$ $\quad y_1 := 3$ $\quad y_2 := 1.5$

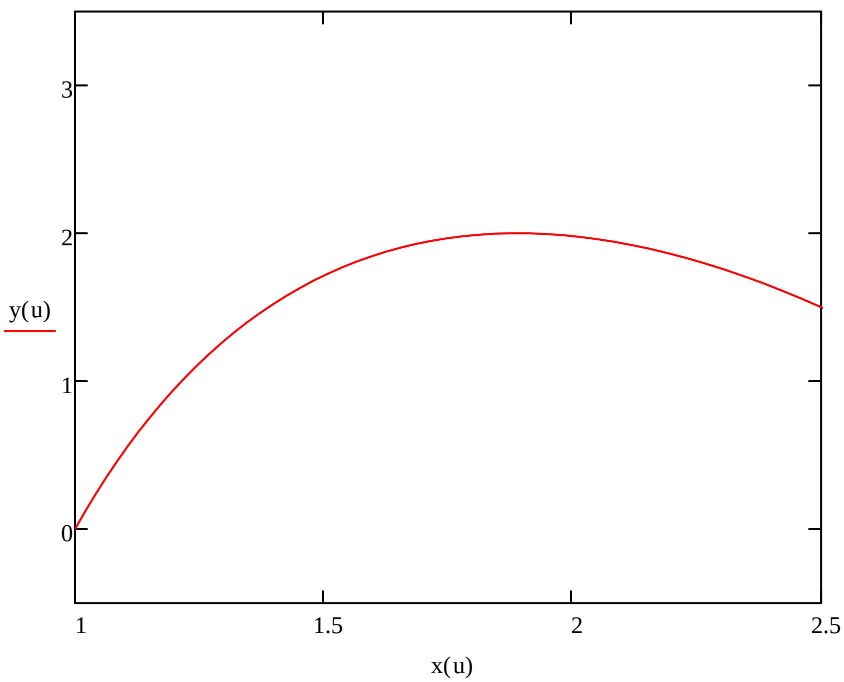
$P_0 := \begin{pmatrix} x_0 & y_0 & 0 \end{pmatrix}$ $\quad P_1 := \begin{pmatrix} x_1 & y_1 & 0 \end{pmatrix}$ $\quad P_2 := \begin{pmatrix} x_2 & y_2 & 0 \end{pmatrix}$

$n := 2$ number of points

$$P(u) := \sum_{i=0}^{n} \left[\frac{n!}{i!\,(n-i)!} u^i (1-u)^{n-i} \cdot P_i \right]$$ curve evaluation

$x(u) := P(u)_{0,0}$ x values as function of u

$y(u) := P(u)_{0,1}$ y values as function of u



$u := 0.9$

$$P_u := \frac{n!}{0!\,(n-0)!}\cdot u^0 (1-u)^{n-0}\cdot P_0 + \frac{n!}{1!\,(n-1)!}\cdot u^1 (1-u)^{n-1}\cdot P_1 + \frac{n!}{2!\,(n-2)!}\cdot u^2 (1-u)^{n-2}\cdot P_2$$

$$x_u := \frac{n!}{0!\,(n-0)!}\cdot u^0 (1-u)^{n-0}\cdot x_0 + \frac{n!}{1!\,(n-1)!}\cdot u^1 (1-u)^{n-1}\cdot x_1 + \frac{n!}{2!\,(n-2)!}\cdot u^2 (1-u)^{n-2}\cdot x_2 = 2.30$$

$$y_u := \frac{n!}{0!\,(n-0)!}\cdot u^0 (1-u)^{n-0}\cdot y_0 + \frac{n!}{1!\,(n-1)!}\cdot u^1 (1-u)^{n-1}\cdot y_1 + \frac{n!}{2!\,(n-2)!}\cdot u^2 (1-u)^{n-2}\cdot y_2 = 1.75$$

$$P_u := (1-u)^2\cdot P_0 + 2\cdot u\cdot(1-u)\cdot P_1 + u^2 P_2$$

$$x := (1-u)^2\cdot x_0 + 2\cdot u\cdot(1-u)\cdot x_1 + u^2 x_2 = 2.305$$

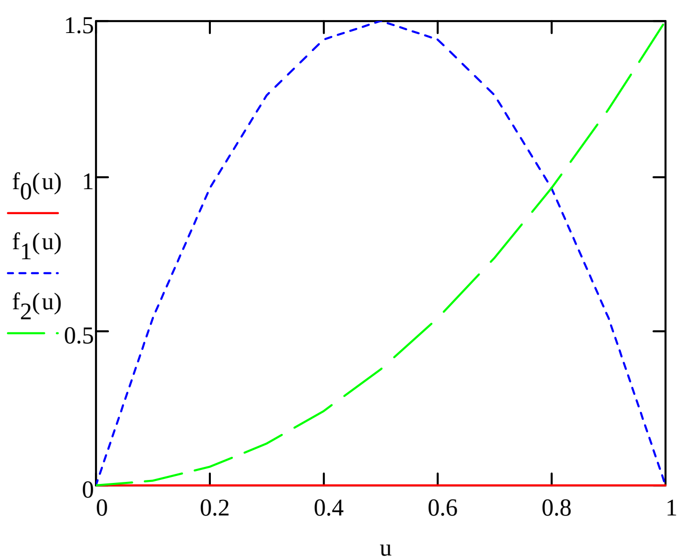
$$y := (1-u)^2\cdot y_0 + 2\cdot u\cdot(1-u)\cdot y_1 + u^2 y_2 = 1.755$$

$u := 0, .1 .. 1$

$f_0(u) := (1-u)^2\cdot y_0$

$f_1(u) := 2\cdot u\cdot(1-u)\cdot y_1$

$f_2(u) := u^2 y_2$



$$f(u) := (1 - u)^2 \cdot y_0 + 2 \cdot u \cdot (1 - u) \cdot y_1 + u^2 y_2$$