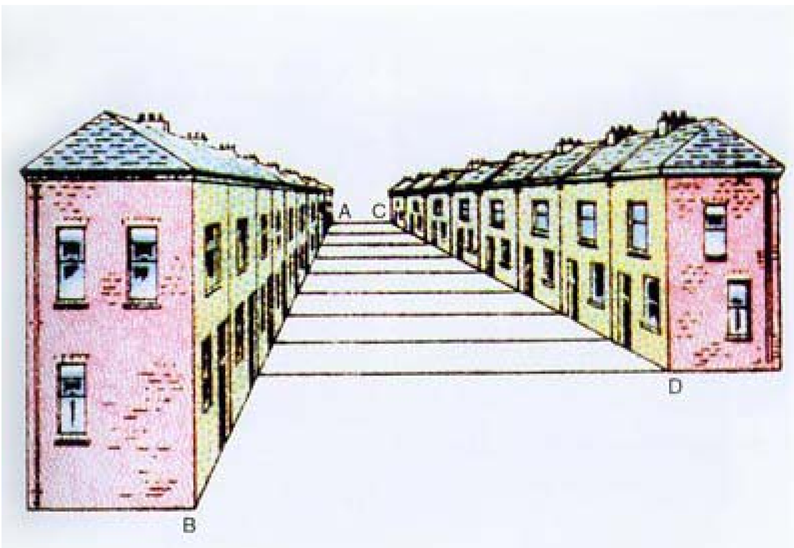
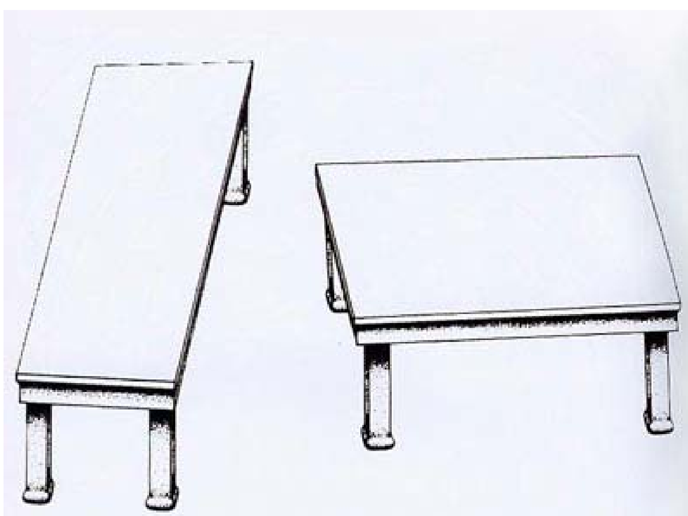
Projection Views





Content

- Coordinate systems
- Orthographic projection
- (Engineering Drawings)

Graphical Coordinator Systems

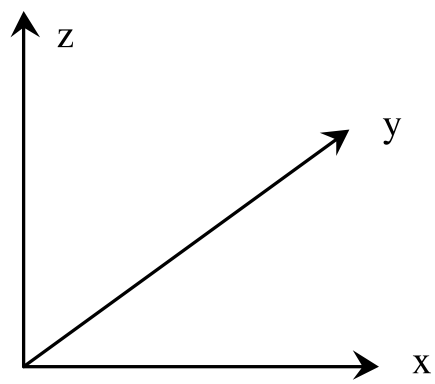
A coordinate system is needed to input, store and display model geometry and graphics.

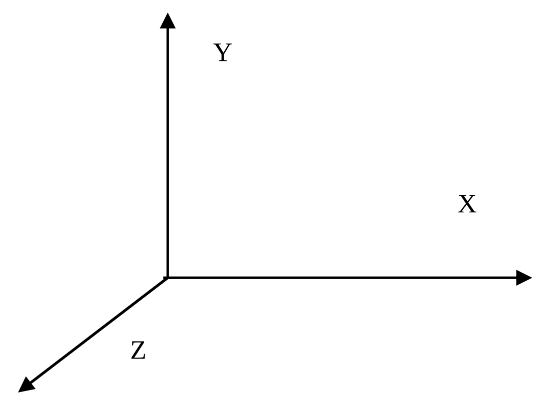
Four different types of coordinate systems are used in a CAD system at different stages of geometric modeling and for different tasks.

Model (or World, Database) Coordinator System

- **The reference space of the model with respect to which all of the geometrical data is stored.**

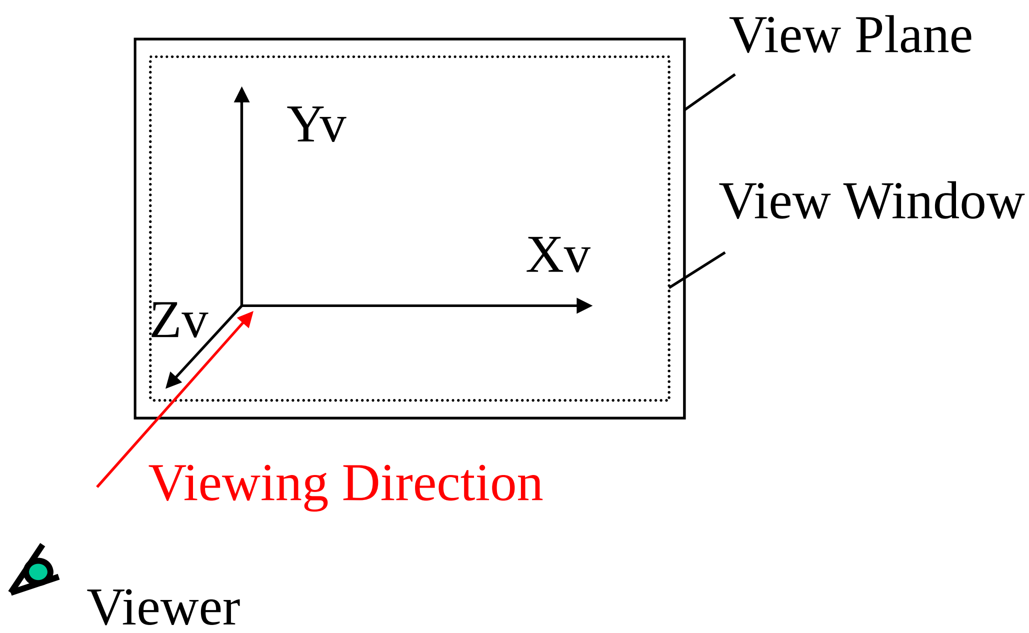
- **It is a Cartesian system which forms the default coordinate system used by a software system.**



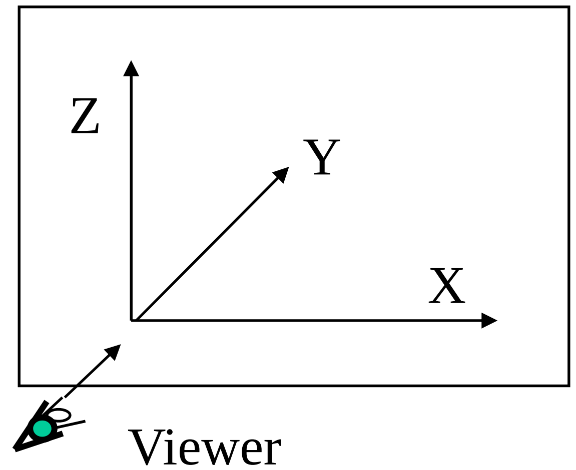


Viewing Coordinate System

Textbook setup - for Parallel Projection, the viewer is at infinity.

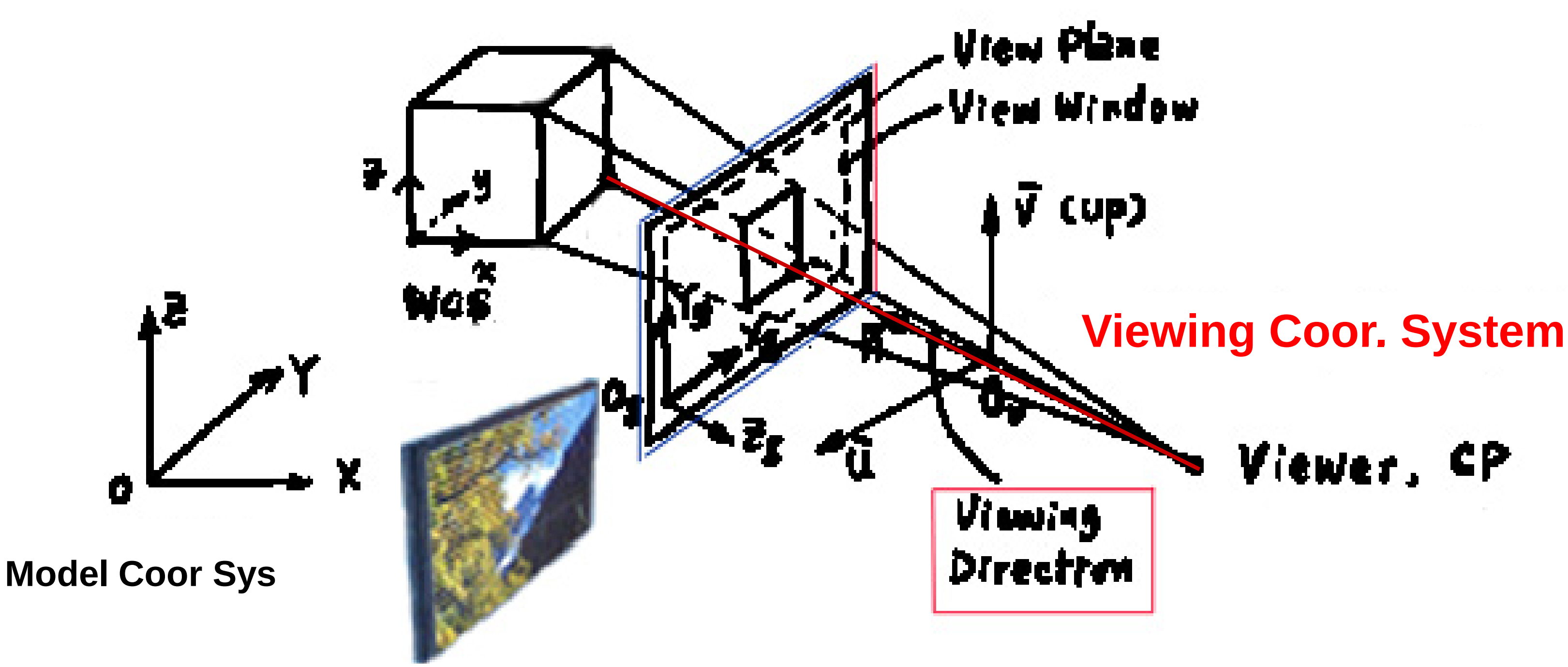


AutoCAD Default Setup

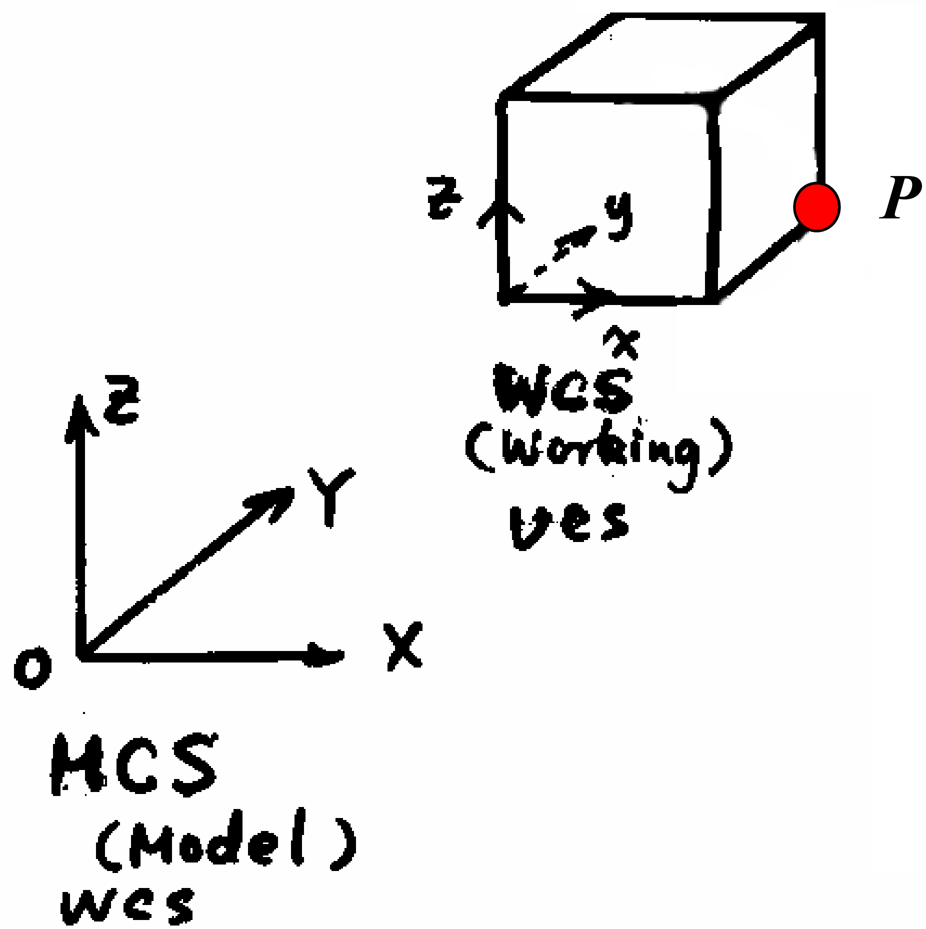


Viewing Coordinate System (VCS)

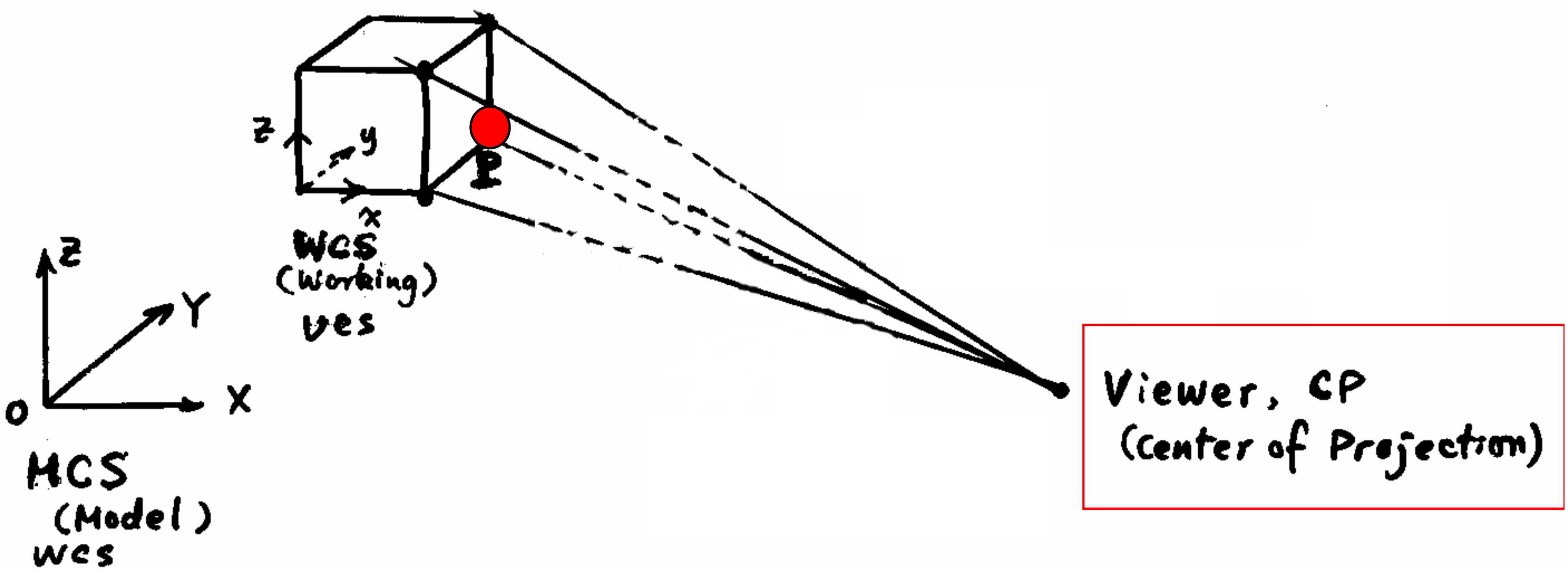
A 3-D Cartesian coordinate system (right hand of left hand) in which a projection of the modeled object is formed. VSC will be discussed in detail under Perspective or Parallel Projections.

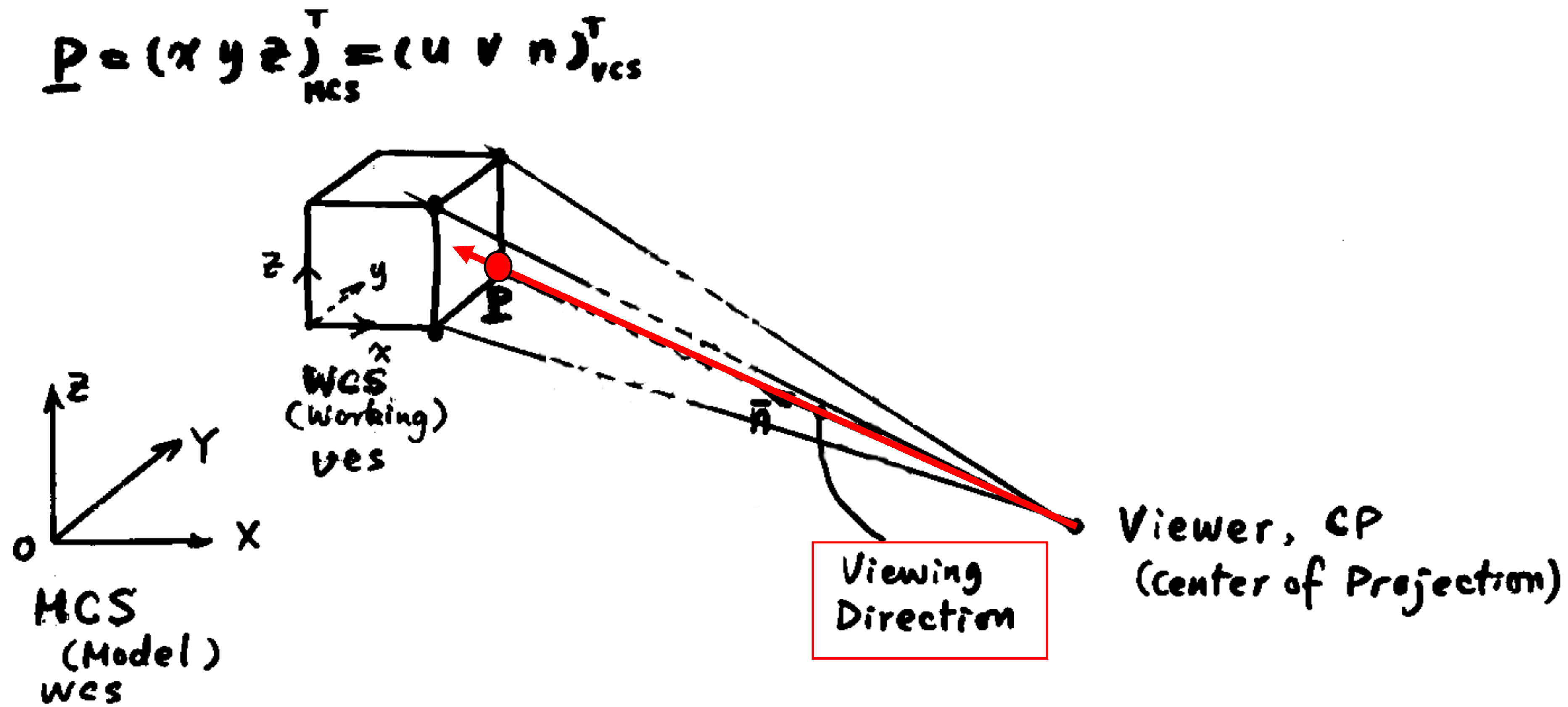


$$\underline{P} = (x\ y\ z)^T$$



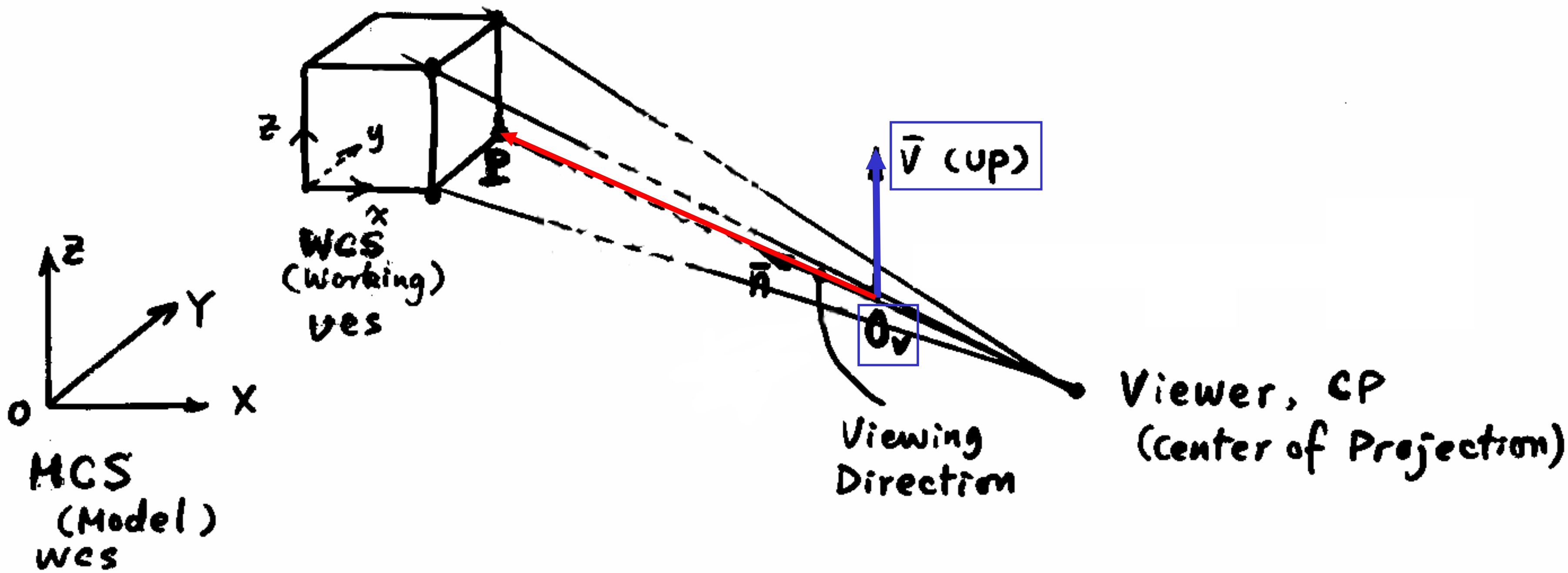
$$\underline{P} = (x\ y\ z)^T_{MCS} = (u\ v\ n)^T_{VCS}$$



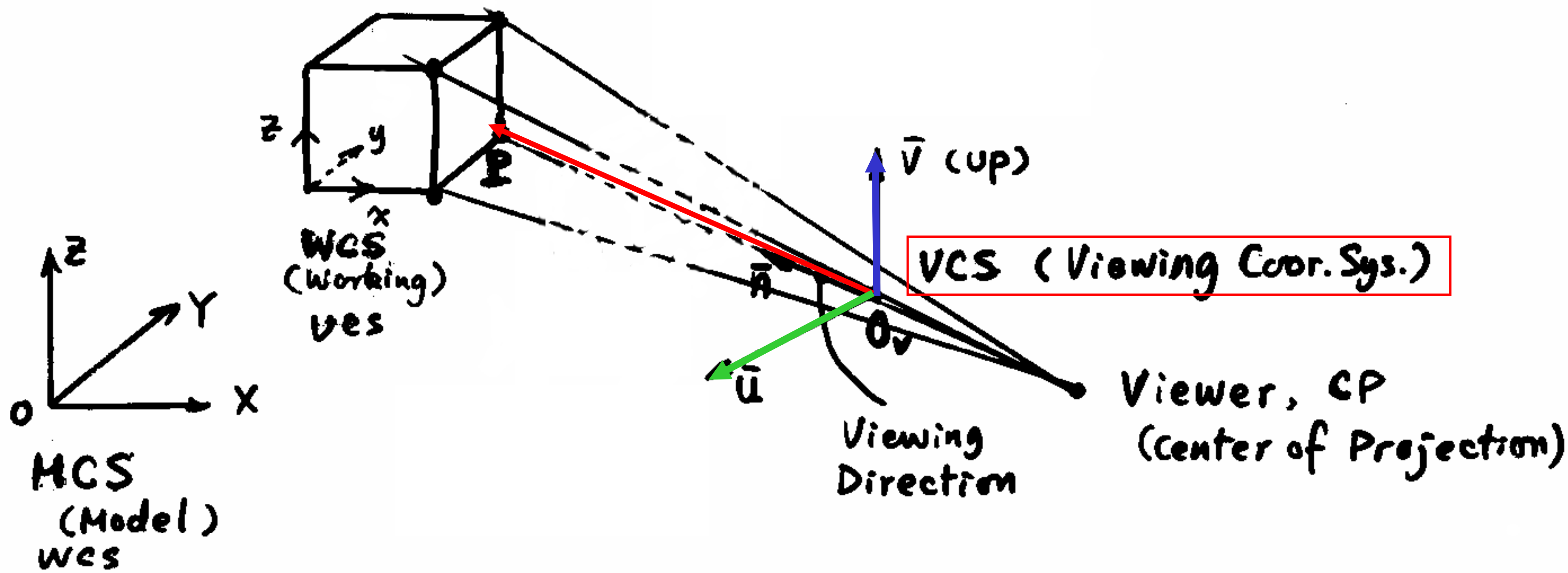


P = (x y z)^T MCS = (u v n)^T VCS
z
y
x
WCS
(Working)
VCS
Z
Y
X
O
MCS
(Model)
WCS
P
n
Viewing Direction
Viewer, CP
(Center of Projection)

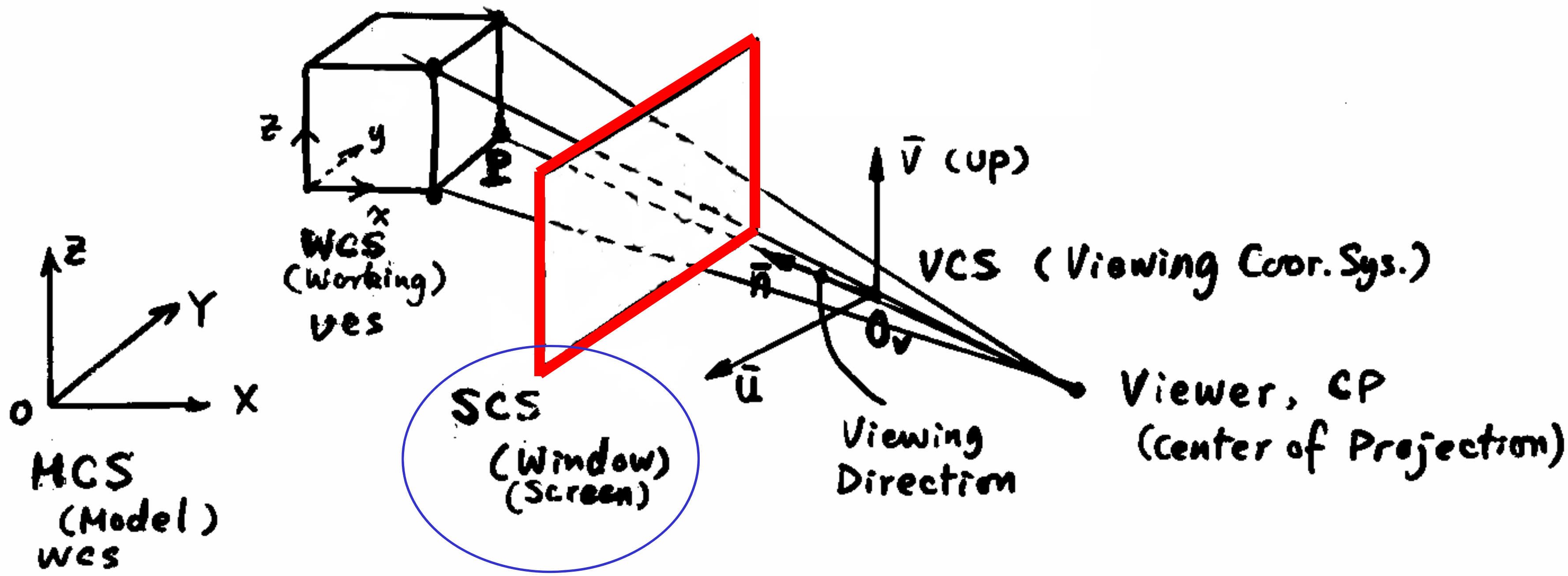
$$\underline{P} = (x\ y\ z)^T_{MCS} = (u\ v\ n)^T_{VCS}$$



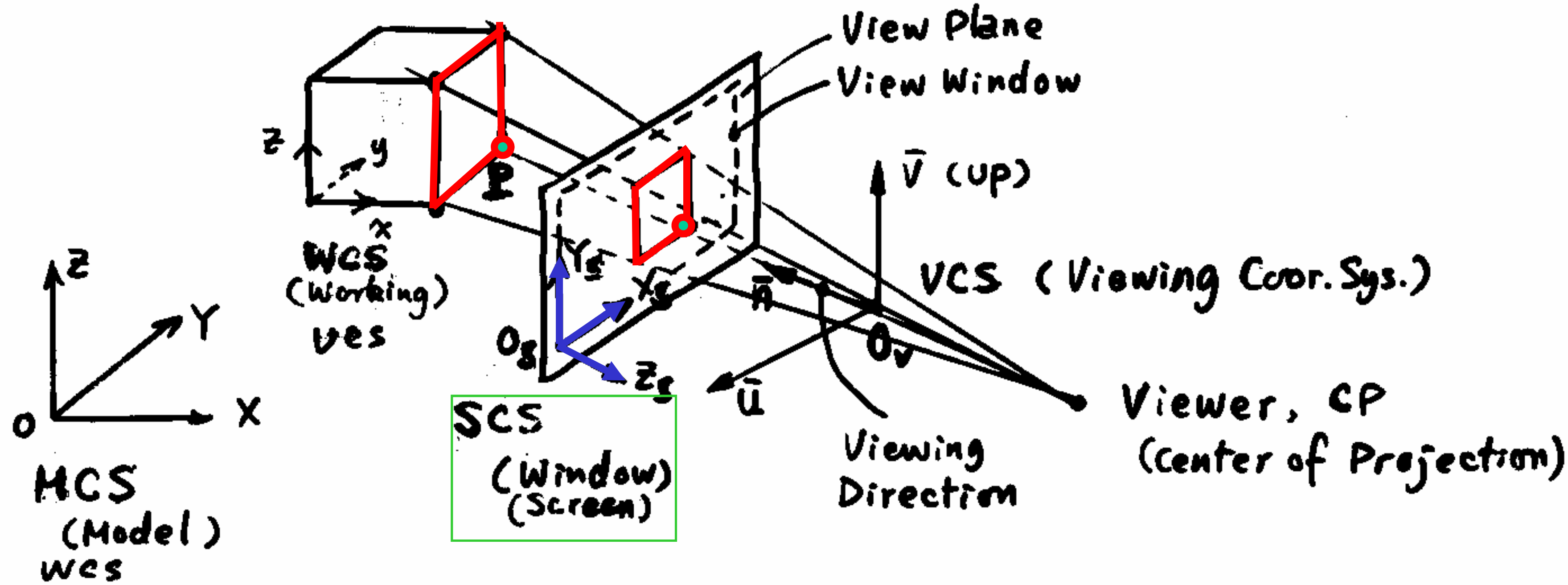
$$\underline{P} = (x\ y\ z)^T_{MCS} = (u\ v\ n)^T_{VCS}$$



$$\underline{P} = (x\ y\ z)^T_{MCS} = (u\ v\ n)^T_{VCS}$$

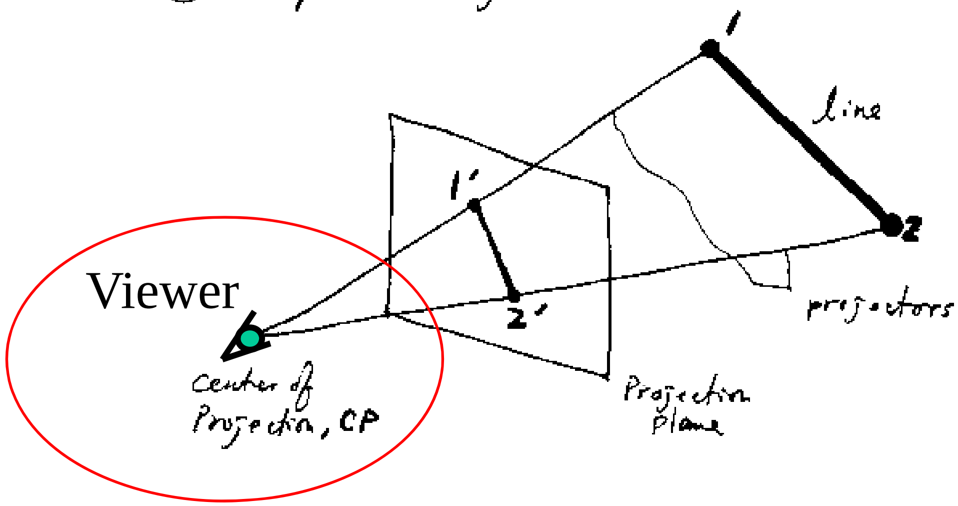


$$\underline{P} = (x\ y\ z)^T_{MCS} = (u\ v\ n)^T_{VCS}$$

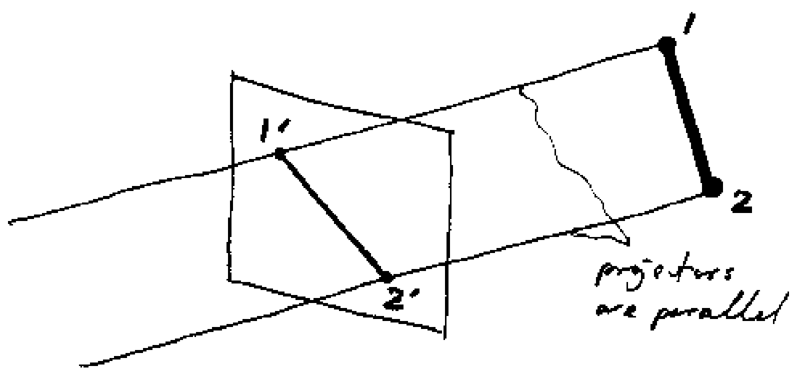


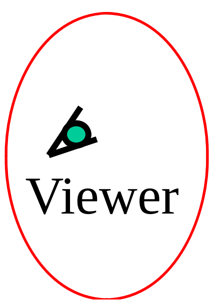
a 3D object.

① Perspective Projection



② Parallel Projection





center of
projection, CP
at infinity

* Parallel lines appear as parallel lines.

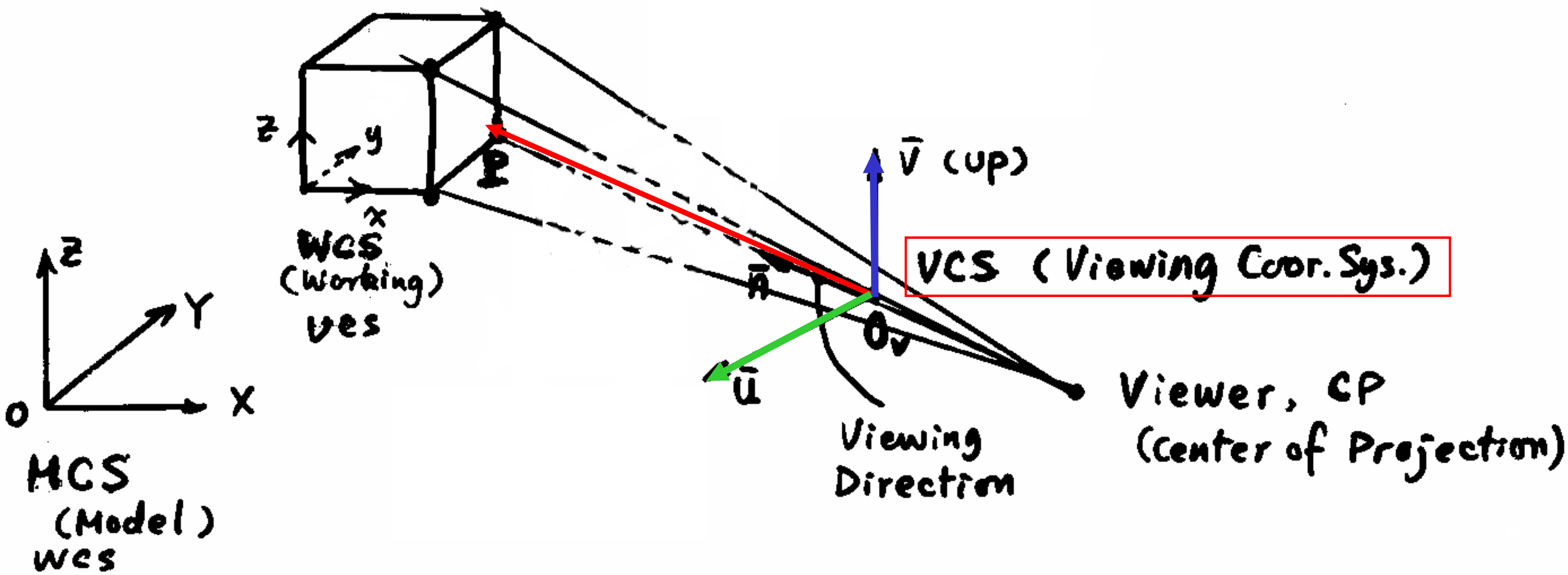
Parallel Projection

- **Preserve actual dimensions and shapes of objects**
- **Preserve parallelism**
- **Angles preserved only on faces parallel to the projection plane**
- **Orthographic projection is one type of parallel projection**

Perspective Projection

- **Doesn’t preserve parallelism**
- **Doesn’t preserve actual dimensions and angles of objects, therefore shapes deformed**
- **Popular in art (classic painting); architectural design and civil engineering.**
- **Not commonly used in mechanical engineering**

$$\underline{P} = (x\ y\ z)^T_{MCS} = (u\ v\ n)^T_{VCS}$$



Geometric Transformations for Generating Projection View

Set Up the Viewing Coordinate System (VCS)

i) Define the view reference point $\mathbf{P} = (P_x,\ P_y,\ P_z)^T$

ii) Define the line of the sight vector $\vec{n}$ **(normalized)**

$$\vec{n} = (N_x,\ N_y,\ N_z)^T \quad \text{and} \quad N_x^2 + N_y^2 + N_z^2 = 1$$

iii) Define the "up" direction

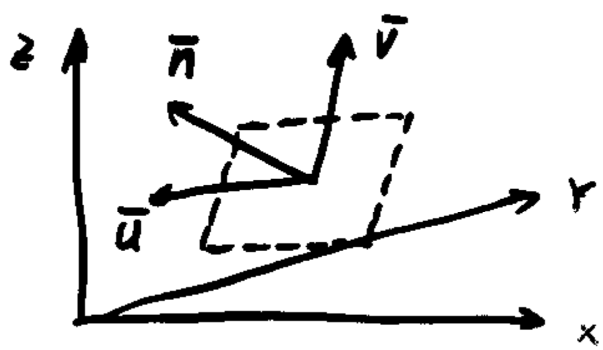
$$\vec{V} = (V_x,\ V_y,\ V_z)^T \perp \vec{n}, \qquad \vec{V} \cdot \vec{n} = 0$$

This also defines an orthogonal vector, $\vec{u} = \vec{V} \times \vec{n}$

$(\vec{u},\ \vec{V},\ \vec{n})$ **forms the viewing coordinates**

Define the View Window in

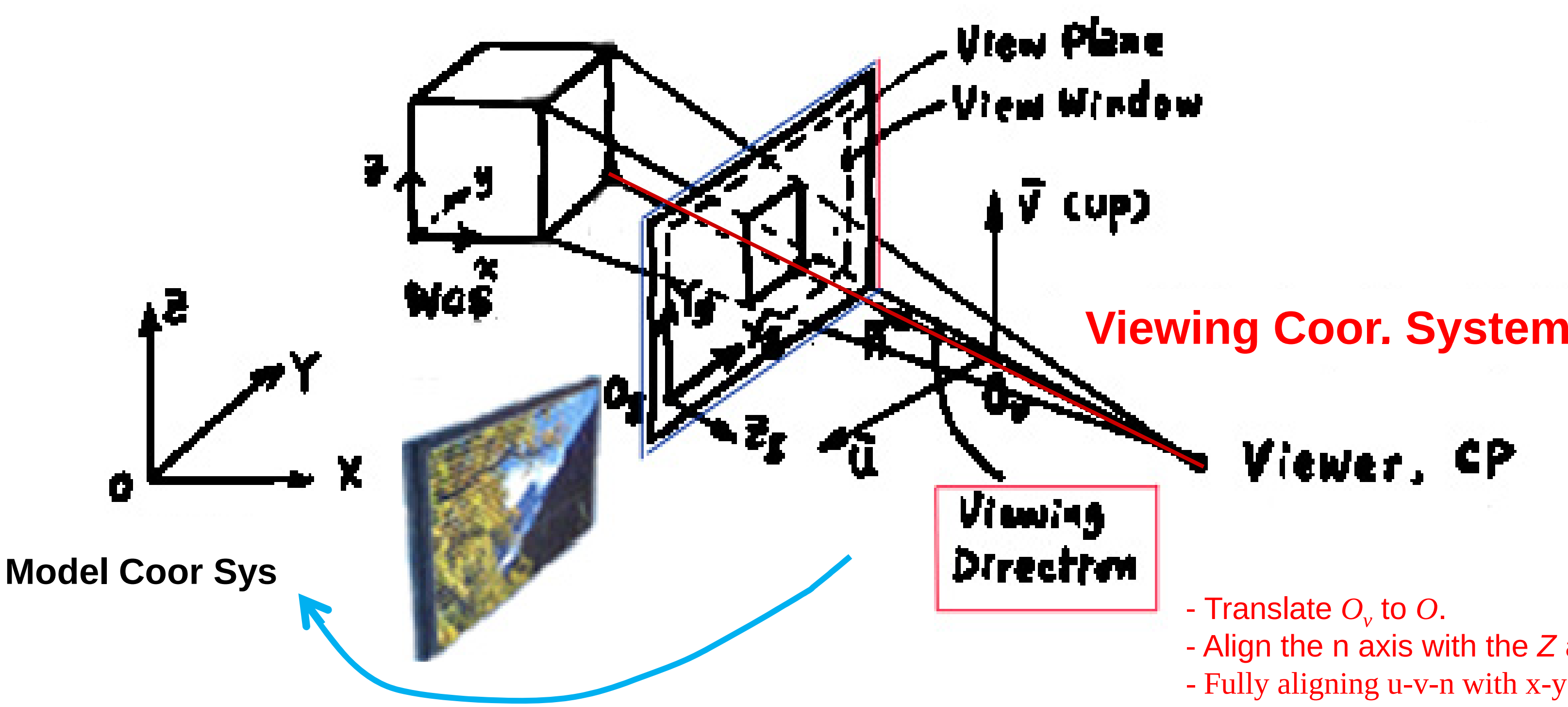
$\overline{u} - \overline{v} - \overline{n}$ **coordinates**



Generating Parallel Projection

Problem: for a given computer model, we know its x-y-z coordinates in MCS; and we need to find its u-v-n coordinates in VCS and X_s-Y_s in WCS.

Getting the u-v-n coordinates of the objects by transforming the objects and u-v-n coordinate system together to fully align u-v-n with x-y-z axes, then drop the n (the depth) component to get X_s and Y_s



- Translate O_v to O.
- Align the n axis with the Z axis.
- Fully aligning u-v-n with x-y-z

Generating Parallel Projection (1)

First transform coordinates of objects into the *u-v-n* coordinates (VCS), then drop the n component. (*n* is the depth)

i.e. Overlapping $u - v - n$ with $x - y - z$

i) Translate O_v to O.

$$[D] = \begin{bmatrix} 1 & 0 & 0 & -0_{vx} \\ 0 & 1 & 0 & -0_{vy} \\ 0 & 0 & 1 & -0_{vz} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

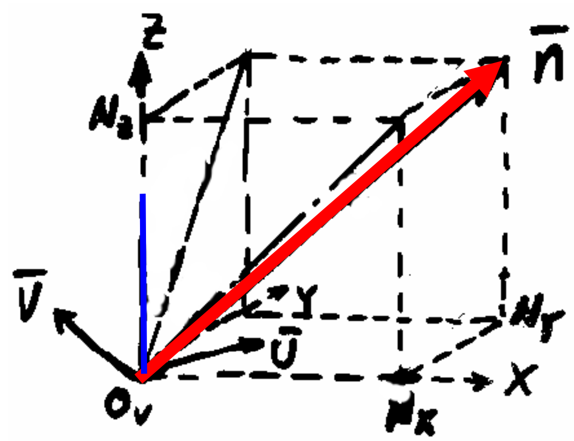
ii) Align the $\vec{n}$ axis with the *Z* axis.

The procedure is identical to the transformations used to prepare for the rotation about an axis.

$$A = N_x, \quad B = N_y, \quad C = N_z$$

$$L = \sqrt{N_x^2 + N_y^2 + N_z^2}$$

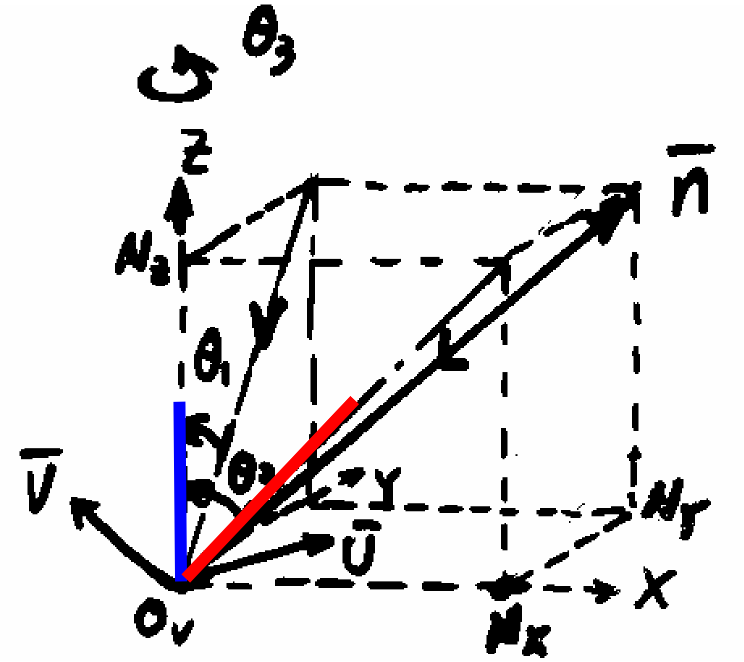
$$V = \sqrt{N_y^2 + N_z^2}$$



Generating Parallel Projection (2)

Rotating θ_1 about *X*: [R]$_x$; and Rotating θ_2 about *Y*: [R]$_y$

Fully aligning u-v-n with x-y-z

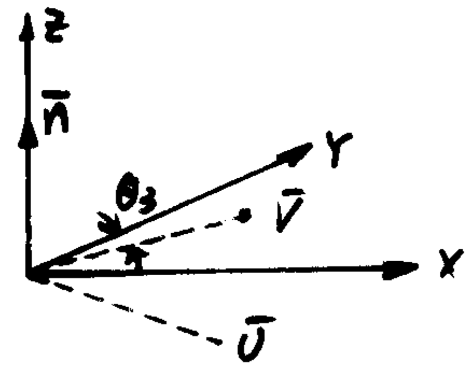


Then, rotate Θ_3 about the Z axis to align $\bar{u}$ with *X* and $\bar{v}$ with *Y*

Generating Parallel Projection (3)

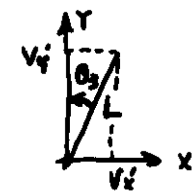
Rotate θ_3 about the Z axis to align $\bar{u}$ with X and $\bar{v}$ with Y

At this point, $\bar{V}$ is given by $(V'_x,\ V'_y,\ 0)^T$ where



$$\begin{pmatrix} V'_x \\ V'_y \\ 0 \\ 1 \end{pmatrix} = [R_y]\,[R_x]\,[D_{O_V,O}] \begin{pmatrix} V_x \\ V_y \\ V_z \\ 1 \end{pmatrix}$$

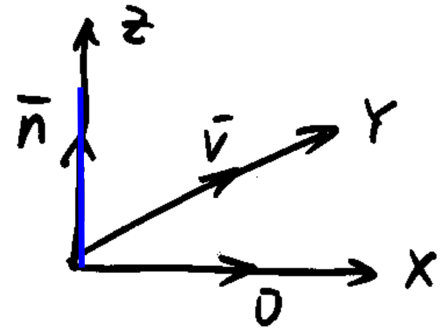
We need to rotate by an angle θ_3 about the Z axis



$$L = \sqrt{V_x'^2 + V_y'^2}, \quad \sin\theta_3 = \frac{V'_x}{L}, \quad \cos\theta_3 = \frac{V'_y}{L}$$

$$[R_z] = \begin{bmatrix} V'_r/L & -V'_x/L & 0 & 0 \\ V'_x/L & V'_y/L & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Result:



Generating Parallel Projection (4)

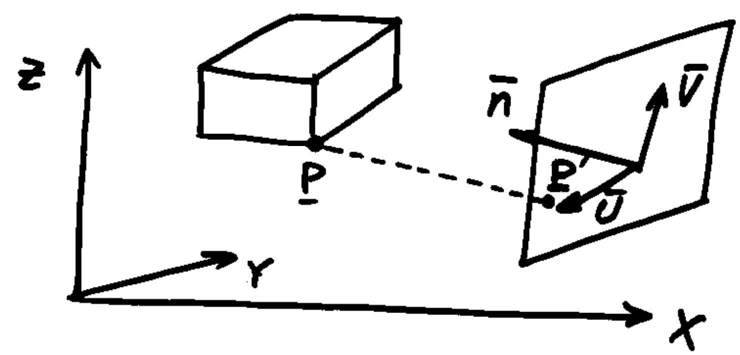
Drop the n coordinate

$$[D_n] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathbf{0} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \begin{pmatrix} u \\ V \\ \mathbf{0} \\ 1 \end{pmatrix} = [D_n] \begin{pmatrix} u \\ V \\ n \\ 1 \end{pmatrix}$$

In summary, to project a view of an object on the UV plane, one needs to transform each point on the object by:

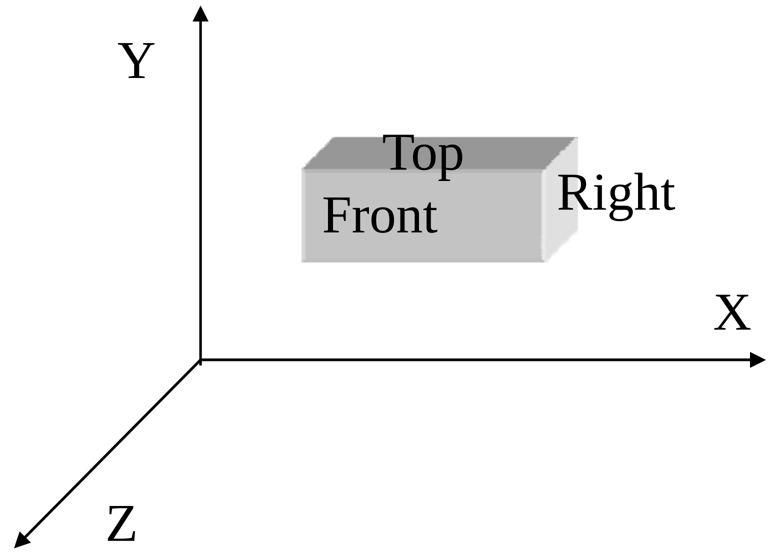
$$[T] = [D_n][R_z][R_y][R_x][D_{o_v,o}]$$

$$\mathbf{P}' = \begin{pmatrix} u \\ V \\ 0 \\ 1 \end{pmatrix} = [T]\mathbf{P} = [T]\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

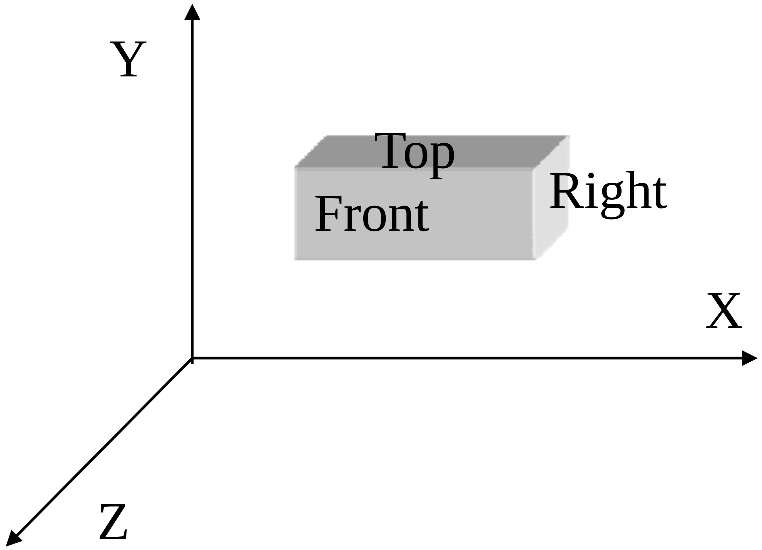


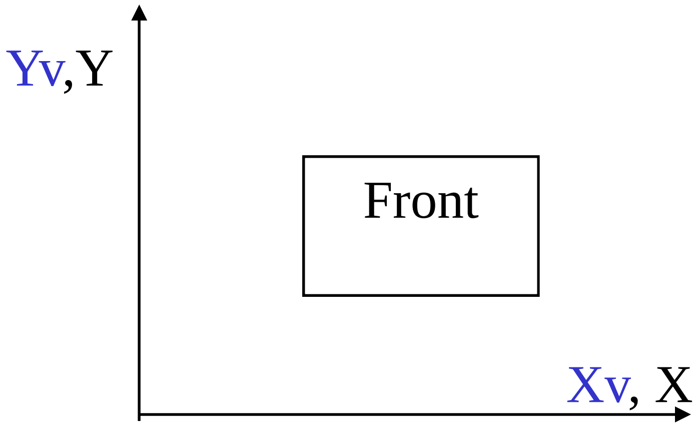
Note: The inverse transforms are not needed! We don't want to go back to x - y - z coordinates.

Orthographic Projection



- Projection planes (Viewing planes) are perpendicular to the principal axes of the MCS of the model
- The projection direction (viewing direction) coincides with one of the MCS axes



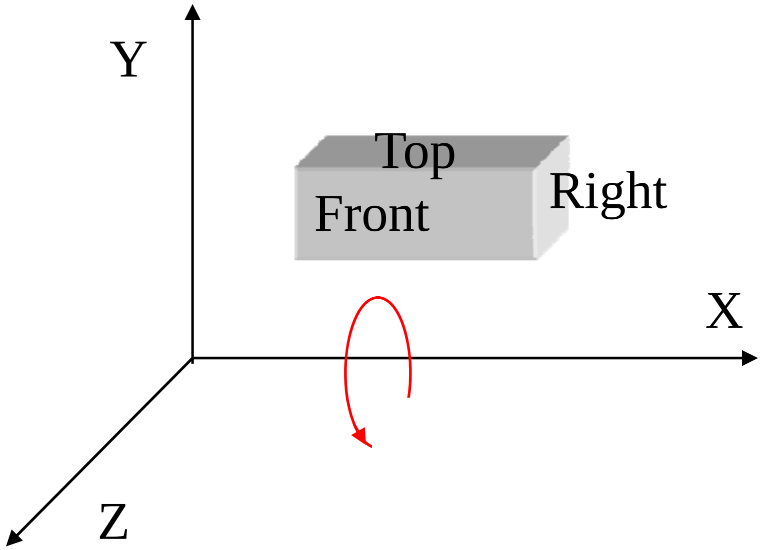


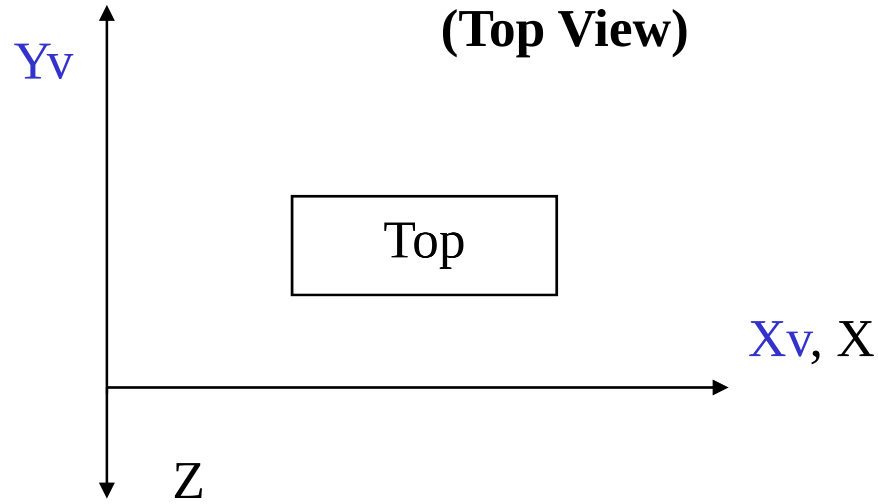
Geometric Transformations for Generating Orthographic Projection (Front View)

$$Pv = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Drop Z

Geometric Transformations for Generating Orthographic Projection (Top View)

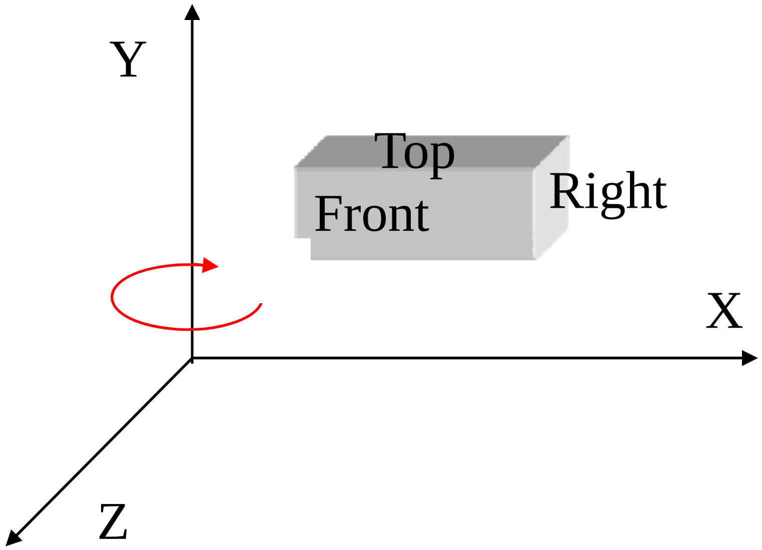


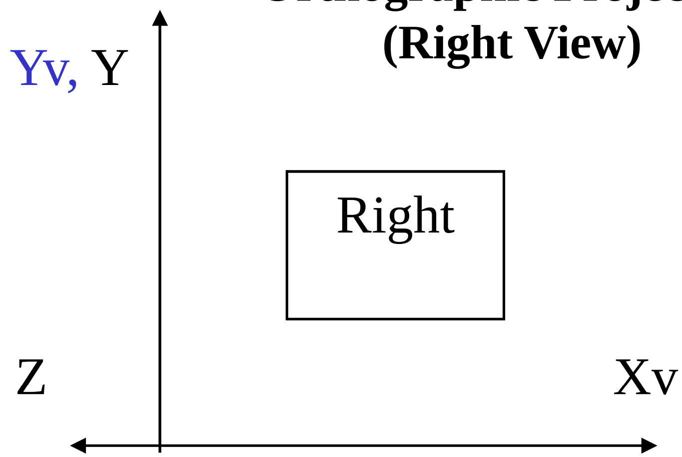


$$P_v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(90°) & -\sin(90°) & 0 \\ 0 & \sin(90°) & \cos(90°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Drop Z $\quad [R]_x^{90}$

Geometric Transformations for Generating Orthographic Projection (Right View)

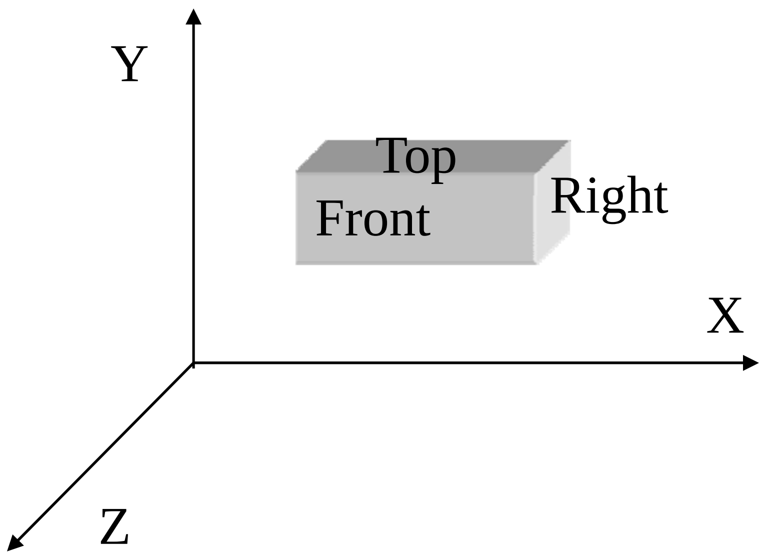


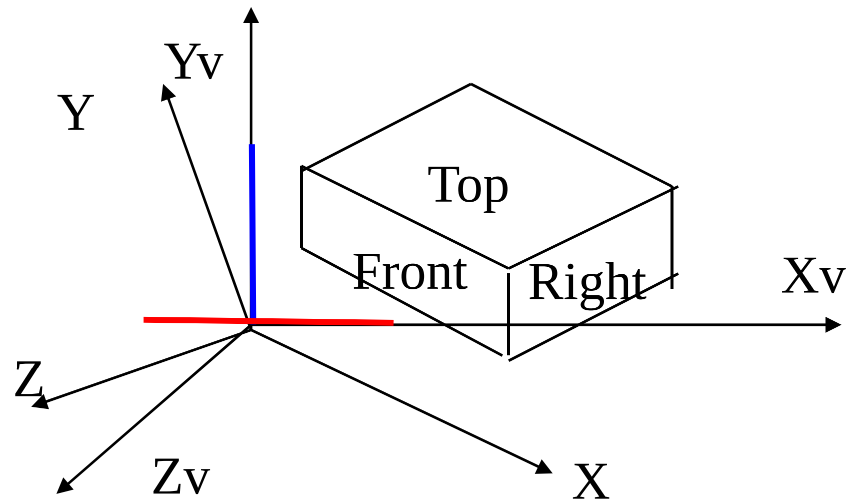


$$P_v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90^\circ) & 0 & \sin(-90^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\left(-90^\circ\right) & 0 & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} P = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Drop Z $[R]_y^{-90}$

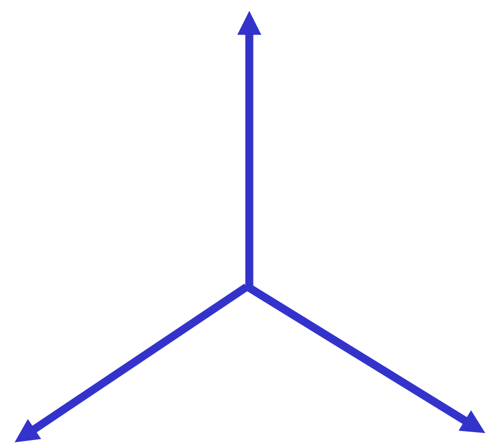
Rotations Needed for Generating Isometric Projection





$$P_v = [R]_x^{\phi}[R]_y^{\theta} P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Isometric Projection: **Equally foreshorten** **the three main axes**



$$\theta = \pm 45°, \phi = \pm 35.26°$$

Other Possible Rotation Paths

- *Rx* --> *Ry*

$$r_x = \pm 45°,\ r_y = \pm 35.26°$$

- *Rz* --> *Ry(Rx)*

$$r_z = \pm 45°, r_{y(x)} = \pm 54.74°$$

- *Rx(Ry)* --> *Rz*

$$r_{y(x)} = \pm 45°, r_z = ANY\ \ ANGLE$$