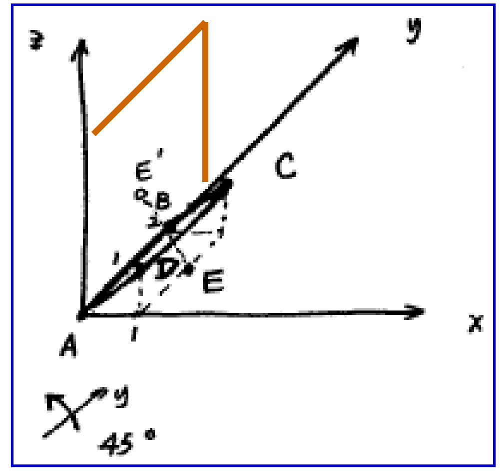
Example Problem 1 – Geometric Transformation

Reflection of a point P (x,y,z) about the x-z plane introduces a new point P'(x,-y,z). A plane is defined by the points A(0,0,0), B(0,2,0), C(1,2,1) and D(1,0,1). Develop a transformation matrix to make reflections of points about this plane and yield results defined in the original x,y,z space. Apply your matrix to the point E at (1,1,0) to obtain its reflection.



① ROTATE ADCB about y 45°

$$R_y = \begin{bmatrix} \cos 45° & 0 & \sin 45° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 45° & 0 & \cos 45° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② Mirror about the yoz plane.

$$M_{zy} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

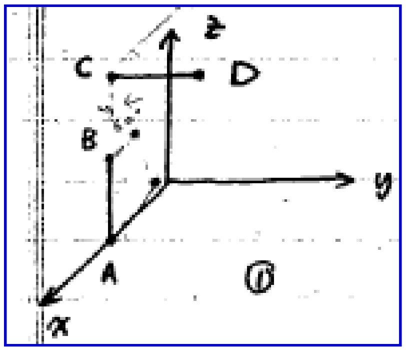
$$C = R_y M_{zy} R^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E' = EC = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \end{array} \begin{array}{c} x \\ Y \\ z \\ \end{array}$$

∴ point E' is at (0 1 1)

Example Problem 2 – Geometric Transformation

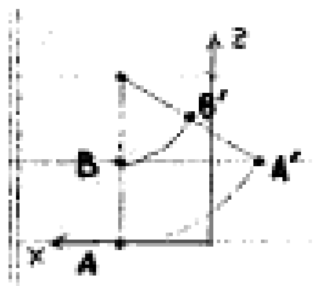
A line connects the point A at (1,0,0) to the point B at (1,0,1). A second line extends from C at (1,0,2) to D at (1,1,2). Rotate line AB about line CD using vector-matrix methods. The rotation should be 60° counter-clockwise if the angle is viewed from the direction of D-to-C.



① CD → Y axis

$$[D] = \begin{bmatrix} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & 1 & -C_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotate AB about CD 60°.



$$[R_y] = \begin{bmatrix} \cos 60^\circ & 0 & \sin 60^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 60^\circ & 0 & \cos 60^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & .866 & 0 \\ 0 & 1 & 0 & 0 \\ -.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

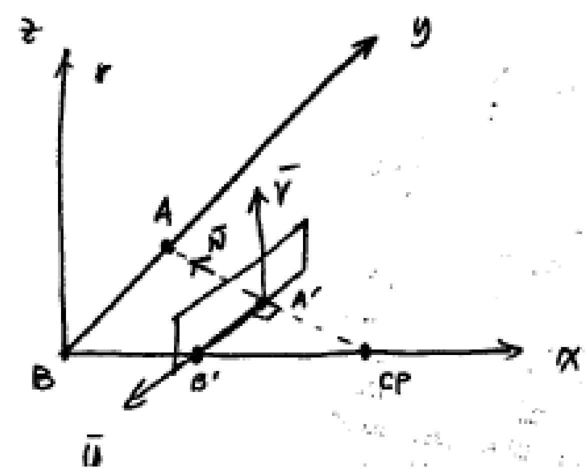
$$[T] = [D]^{-1}[R_y][D] = \begin{bmatrix} 0.5 & 0 & 0.866 & -1.232 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 1.866 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

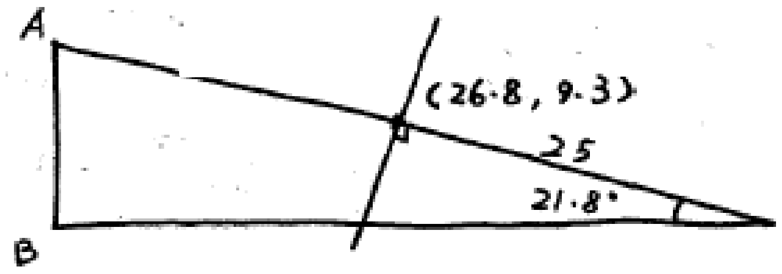
$$B' = [T]B = \begin{bmatrix} 0.134 \\ 0 \\ 1.5 \\ 1 \end{bmatrix}$$

$$A' = [T]A = \begin{bmatrix} -0.732 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

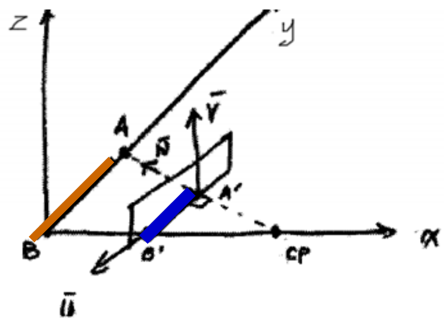
Example Problem 3 - Generation of Projection View

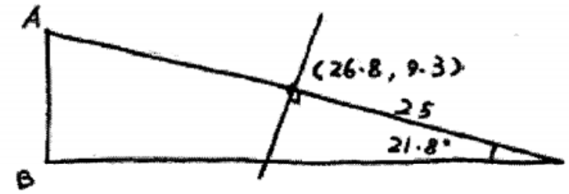
Point A is at (0,20,0) and point B is at (0,0,0). The line AB is to be viewed from (50,0,0) looking directly at point A. The display surface is 25 units from the viewer (between the viewer and the object). What is the image of the line AB on the display coordinates? Use a methodical matrix approach of a type described in class.



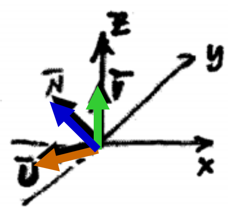


Example Problem 3 Generation of Projection View

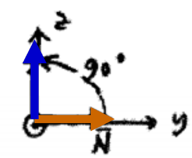




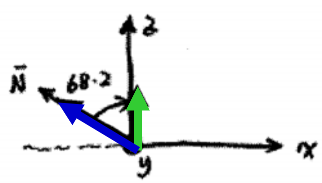
$$T = \begin{bmatrix} 1 & 0 & 0 & -26.8 \\ 0 & 1 & 0 & -9.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



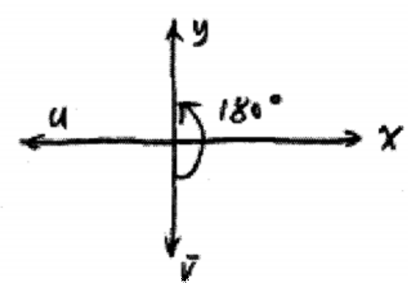
$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90^\circ & -\sin 90^\circ & 0 \\ 0 & \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$



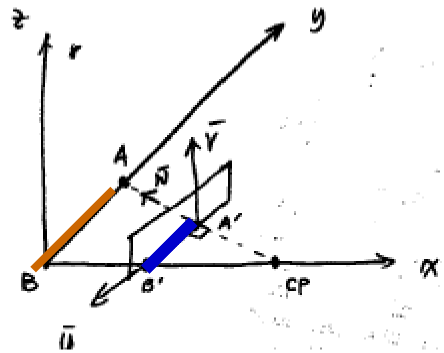
$$R_y = \begin{bmatrix} \cos 68.2 & 0 & \sin 68.2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 68.2 & 0 & \cos 68.2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

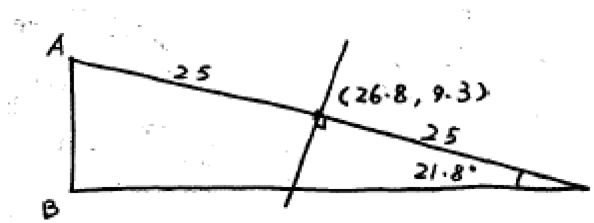


$$R_z = \begin{bmatrix} \cos 180^\circ & -\sin 180^\circ & 0 & 0 \\ \sin 180^\circ & \cos 180^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Example Problem 3 Generation of Projection View





$$R_z R_y R_x T = \begin{bmatrix} -0.37 & -0.93 & 0 & 10.8 \\ 0 & 0 & 1 & 0 \\ -0.93 & 0.37 & 0 & 24.6 \\ 10.8 & 0 & 24.6 & 1 \end{bmatrix}$$

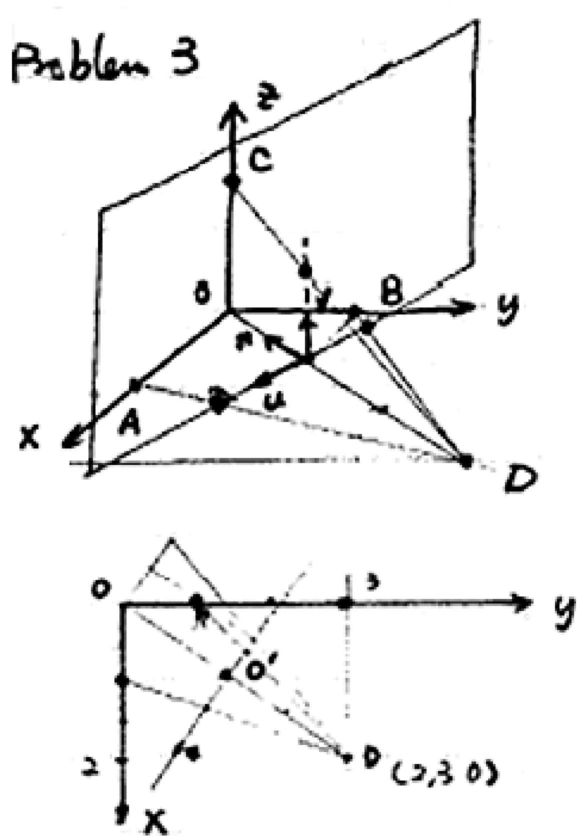
$$\underline{P}' = R_z R_Y R_x T \underline{P}$$

$$D_{pp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P'' = D_{pp} P' = [10 \quad 0 \quad 0 \quad 1]^T$$

Example Problem 4 Generation of Projection View

If the three point $A=(1,0,0)^T$, $B=(0,1,0)^T$ and $C=(0,0,1)^T$ is viewed from the point $D=(2,3,0)^T$ via looking at the origin $O=(0,0,0)^T$, and using a projection plane 2 units away from point D (between O and D). Determine the **parallel** projections (u and v coordinates) for point A, B and C.



① Geometry.

$\tan\theta = \frac{2}{3}$, $\theta = 33.69°$

$X_{o'} = 2 - 2\sin 33.69° = 0.89$

$Y_{o'} = 3 - 2\cos 33.69° = 1.336$

① $O' \to O$ $[0.89 \;\; 1.336 \;\; 0]^T \to 0.$

② R_z: $33.69°$

③ R_x: $-90°$

④ CP: $[\underset{u_p}{0} \;\; \underset{v_p}{0} \;\; \underset{n_p}{-2}]^T$

Example Problem 4 Generation of Projection View

1) $$[T]_{0-0'} = \begin{bmatrix} 1 & 0 & 0 & -0.89 \\ 0 & 1 & 0 & -1.336 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2) $$[R]_z^{33.69^\circ} = \begin{bmatrix} \cos 33.69^\circ & -\sin 33.69 & 0 & 0 \\ \sin 33.69 & \cos 33.69 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) $$[R]_x^{-90^\circ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example Problem 5 – Half Space, Solid Modeling

Problem 1

For the solid model shown in Figure 1, give its solid model representation using appropriate "half-space" definitions and logical operations.

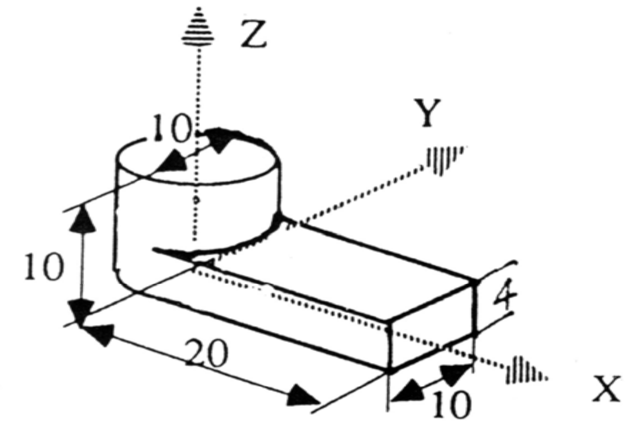
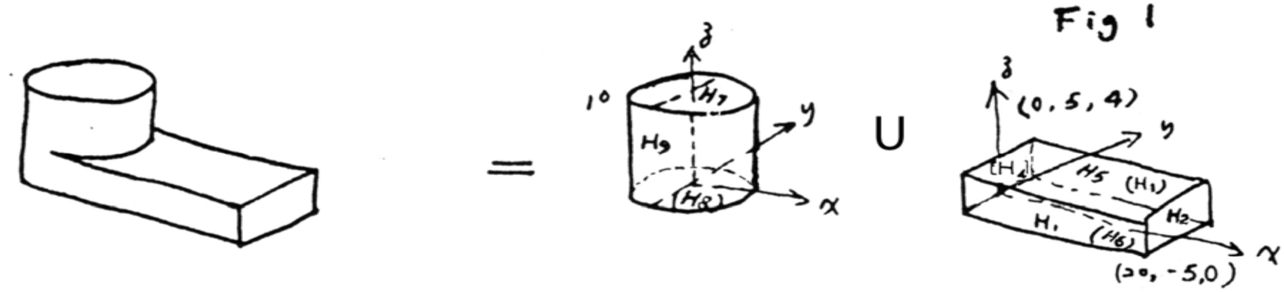


Fig 1



$H_1 : \{(x,y,z) : y > -5\}$

$H_2 : \{(x,y,z) : x < 20\}$

$H_3 : \{(x,y,z) : y < 5\}$

$H_4 : \{(x,y,z) : x > 0\}$

$H_5 : \{(x,y,z) : z < 4\}$

$H_6 : \{(x,y,z) : z > 0\}$

$H_7 : \{(x,y,z) : z < 10\}$

$H_8 = H_6 :$

$H_9 : \{(x,y,z) : x^2 + y^2 < 25\}$

or:

H	pt 1	pt 2
H_1	(20 -5 0)	(20 -4 0)
H_2	(20 -5 0)	(19 -5 0)

$$H = (H_1 \cup H_2 \cup H_3 \cup H_4 \cup H_5 \cup H_6) \cup (H_6 \cup H_7 \cup H_9)$$

Intersections Union Intersections

Example Problem 6 – Optimization

A design optimization problem is formulated as

$$\min_{x_1, x_2} \; x_1^2 + 4x_1 + x_2^2 + 2x_2 + 5$$

$$\begin{aligned} s.t. \qquad x_2 &= x_1 + 1 \\ x_1 + x_2 &\le 2 \\ x_1 \ge 0;\; x_2 &\ge 0 \end{aligned}$$

- Rearrange this formulation into the standard form for design optimization
- Find the close-form solution of the unconstrained optimization with no design constraints
- Graphically illustrate the solution of the design optimization and identify the solution of the original constrained optimization problem

Example Problem 6 – Optimization

Rearrange this formulation into the standard form for design optimization

$$\min_{x_1, x_2} \; x_1^2 + 4x_1 + x_2^2 + 2x_2 + 5$$

$$s.t. \; -x_1 + x_2 - 1 = 0$$

$$x_1 + x_2 - 2 \le 0$$

$$-x_1 \le 0$$

$$-x_2 \le 0$$

$$\bar{x} = [x_1, x_2]^T$$

Example Problem 6 – Optimization

Find the close-form solution of the unconstrained optimization with no design constraints

$$f(x_1, x_2) = x_1^2 + 4x_1 + x_2^2 + 2x_2 + 5$$

$$= x_1^2 + 4x_1 + \left(x_1 + 1\right)^2 + 2\left(x_1 + 1\right) + 5$$

$$= 2x_1^2 + 8x_1 + 8 = 2\left(x_1 + 2\right)^2$$

$$\frac{df(x_1, x_2)}{dx_1} = x_1 + 2 = 0; \;\; x_1^* = -2; \;\; x_2^* = x_1^* + 1 = -1$$

$$\bar{x}^* = [x_1^*, x_2^*]^T = [-2, -1]^T$$

This is the solution without considering the constraints!

Example Problem 6 – Optimization

- Graphically illustrate the solution of the design optimization and identify the solution of the original constrained optimization problem