Lecture 10:
Piezoresistivity

Piezoresistivity Overview

- Design of Piezoresistive Sensors with Micro-Beams
- Stress Distribution in Thin Plates
- Design of Piezoresistive Sensors with Thin Plates
The maximum strain in a beam occurs in the location where the moment is highest.

Recall from Lecture 5, the stress for a beam in a state of ‘pure bending’ is:

\[
\sigma = \frac{Mc}{I}
\]

where: \( M = \text{moment} (N \cdot m) \)

\( c = \text{distance to neutral axis} (m) \)

\( I = \text{moment of inertia} (m^4) \)

Since stress and strain are related by: \( \sigma = E\varepsilon \), the strain at that point in the beam is:

\[
\varepsilon = \frac{Mc}{EI}
\]

### Four Cases of Maximum Stress and Strain
(for cantilever beams of rectangular cross-section)

Consider the following points on a cantilever beam:
Four Cases of Maximum Stress and Strain
(for cantilever beams of rectangular cross-section)

Case 1: Beam in Pure Tension:

Stress or strain at all points is equal

\[
\sigma = \frac{F}{A} = \frac{F}{wt} \quad \text{for all points}
\]

\[
\varepsilon = \frac{F}{AE} = \frac{F}{wtE} \quad \text{for all points}
\]

Case 2: Beam in Pure Bending about the x-axis:

Stress or strain is greatest at points A, B, C, E:

\[
\sigma_{(\text{max})} = \frac{Mc}{I} = \frac{Ft}{\left(\frac{wt^3}{12}\right)} = \frac{6FL}{wt^2}
\]

\[
\varepsilon_{(\text{max})} = \frac{Mc}{IE} = \frac{6FL}{wt^2E}
\]
Four Cases of Maximum Stress and Strain
(for cantilever beams of rectangular cross-section)

Case 3: Beam in Pure Bending about the z-axis:

Stress or strain is greatest at:
points A (compression),
points C, D, E (tension):

\[
\sigma_{(\text{max})} = \frac{Mc}{I} = \frac{FLw}{2} = \frac{6FL}{tw^2}
\]

\[
\varepsilon_{(\text{max})} = \frac{Mc}{IE} = \frac{6FL}{tw^2E}
\]

Case 4: Beam in Torsion (twist) about the y-axis:

‘Shear stress’ or ‘shear strain’ is greatest at points: B, G
(assuming pure torsion)

See Table in Lecture 5, page 22 for equations for shear strain
at either point B or G (or any point between them along a line)
Example: Resistance Change due to Applied Force

Question: What is the percentage change in resistance, given the applied load $F$?

See Class Notes for Solution

Stress in Membranes or Thin Plates

- Micro-membranes and thin plates are widely used as pressure sensors for fluids or gases.
- By determining the state of strain of a membrane, we can find the pressure applied to it.
- The stress and strain analysis of membranes requires a ‘fairly complicated’ 2-dimensional analysis, unlike the simple 1-dimensional equations for beam bending.
- For this course, we will only consider a general rectangular membrane shape, subjected to a uniform applied pressure, as shown on the next page.
We assume that a ‘uniform pressure’ is applied to the top surface.

Recall the FEM analysis from Lecture 10:

The following ‘empirical’ formula can be used, along with Table-L11, on page 14 of these notes.

(NOte: Table 6.10 in the textbook is incorrect.)

The formula for maximum stress (at edges) is:

$$\sigma_{(\text{max})} = \frac{\beta pb^2}{t^2}$$

where:

- $p$ – pressure ($N/m^2$)
- $b$ – short edge length
- $t$ – thickness
Stress and Displacement of Square Membranes

The formula for stress at the center of the plate is:

$$\sigma_{(center)} = \frac{\beta_2 pb^2}{t^2}$$

where: $$\beta_2$$ – from table L14

The formula for displacement of the plate in the center is:

$$\delta_{(center)} = \frac{\alpha pb^4}{Et^3}$$

where: $$\delta_{(center)}$$ – maximum deflection at center

$$\alpha$$ – from table L14

$$E$$ – Young’s modulus

We can find the strain by using the relation:

$$\varepsilon = \frac{\sigma}{E}$$

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Stress and Displacement of Square Membranes

Table L11: Constants for bending of Rectangular Plate under a Uniform pressure load:

<table>
<thead>
<tr>
<th>a/b</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>β1</td>
<td>0.3078</td>
<td>0.3834</td>
<td>0.4356</td>
<td>0.4680</td>
<td>0.4872</td>
<td>0.4974</td>
<td>0.5000</td>
</tr>
<tr>
<td>β2</td>
<td>0.1386</td>
<td>0.1794</td>
<td>0.2094</td>
<td>0.2286</td>
<td>0.2406</td>
<td>0.2472</td>
<td>0.2500</td>
</tr>
<tr>
<td>α</td>
<td>0.0138</td>
<td>0.0188</td>
<td>0.0226</td>
<td>0.0251</td>
<td>0.0267</td>
<td>0.0277</td>
<td>0.0284</td>
</tr>
</tbody>
</table>

Example 2: Resistance Change of Pressure Sensor

Consider a rectangular membrane fabricated with silicon crystal using bulk micromachining. A doped resistor has been fabricated into one edge, as shown below:

Question: If the membrane is 4 um thick, and there is a pressure difference of \((P_2 - P_1)\) 1000 kPa from one side to the other, what is the new resistance?

See Class Notes for Solution
Case Study: Multi-Axis Piezoresistive Tactile Sensor

A tactile sensor can be made using the piezoresistive effect, using the following design:

Figure 6.18. Multi-Axis Tactile Sensor, [Chang Liu]

Case Study: Piezoresistive Flow Shear Stress Sensor

A shear flow sensor to measure fluid flow can be made using the piezoresistive effect, as follows:

Figure 6.19. Flow Shear Sensor, [Chang Liu]