Lecture 7: Thermal Sensors & Actuators

Thermal Sensing and Actuation
Overview

Thermal Sensors and Actuators
Example of Inkjet
Example of Thermal Bimorph
Basics of One-Dimensional Heat Transfer
Example of Heat Transfer when Boiling Water
Modeling Heat Transfer as ‘thermal circuits’
Thermal Energy Storage
Heating and Cooling Rates
Thermal Sensing and Actuation

Thermal Energy (Heat) can cause physical changes to materials such as:
- Thermal expansion/contraction (size)
- Electrical resistance
- Optical radiation emission (light)
- Phase change (fluid-gas)

MEMS devices can transduce thermal energy to create:

- Microactuators:
  - Thermal bimorph
  - Fluid dispensers

- Micro-sensors:
  - Thermal bimorph
  - Micro thermocouples
  - Thermo resistive sensors

Example of Ink Jet Thermal Actuator

Inkjet printers operate on the ‘Phase change’ aspect of micro-thermal actuation.

Consider the example below:

When current is applied to the polysilicon strip, ohmic heating occurs.

This causes local heating of fluid adjacent to the heater, causing a phase change of the ink from fluid to gas.

Example of Electro-Thermal Actuator

Movie of micro-electro-thermal actuators:
Example of Electro-Thermal Actuator

Operational details of electro-thermal actuator:

The different areas, given the constant current, \( i \), causes a different amount of heating in each area.

This in turn causes a different amount of thermal expansion, which can be utilized to produce motion of the tip.

This lateral motion can be harnessed to do useful work.
There are four modes of heat transfer

(1) Thermal conduction:
- flow of heat through solid matter

\[ q_{\text{cond}}'' = -\kappa \frac{dT}{dx} \]

where: \( q_{\text{cond}}'' \) = heat flux in units of \( \frac{\text{Watts}}{\text{m}^2} \)

\( \kappa \) = (kappa) thermal conductivity

\( T \) = Temperature

\( x \) = 1-D coordinate direction
(2) Natural thermal convection:
- flow of heat from a surface into a stationary fluid

\[ q_{\text{conv}}^* = h(T_s - T_\infty) \]

where:
- \( q_{\text{conv}}^* \) = heat flux in units of \( \frac{\text{Watts}}{m^2} \)
- \( T_s \) = Body surface temperature
- \( T_\infty \) = Ambient fluid/gas temperature
- \( h \) = convective heat transfer coefficient \( \frac{\text{Watts}}{m^2K} \)

(3) Forced thermal convection:
- flow of heat from a surface into a moving fluid

\[ q_{\text{conv}}^* = h(T_s - T_\infty) \]
Basics of Heat Transfer

(4) Radiation:
- loss of heat via electromagnetic radiation through air or vacuum from a surface

\[ E = \varepsilon \sigma T_r^4 \]

where: \( E \) = Emissive energy/unit area \( \left( \frac{Watts}{m^2} \right) \)

\( \varepsilon \) = radiative emissivity

\( \sigma \) = Stefan - Boltzmann Const.

\( T_r \) = Body surface temperature in K (Kelvin)

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Heat Transfer

The “driving force” for heat flow is temperature difference
- i.e. Heat will only flow from one point to another if there is a temperature difference between these two points.
- Heat will only flow from High Temp ---> Low Temp.

A useful analogy between heat (thermal) and current (electrical) is:

<table>
<thead>
<tr>
<th>Thermal Domain</th>
<th>Electrical Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>heat flux, ( q'' ) (watts/m^2)</td>
<td>current density, ( J = i/m^2 ) (amps/m^2)</td>
</tr>
<tr>
<td>Temp Differ., ( \Delta T ) (degrees Kelvin)</td>
<td>Potential Difference, ( V ) (Volts)</td>
</tr>
<tr>
<td>Thermal Resistance, ( R_{th} ) (K/Watt)</td>
<td>Resistance, ( R ) (ohms)</td>
</tr>
</tbody>
</table>
Example of Heat Transfer: Boiling Pot of Water

Consider the Diagram below:

1. Conduction in Pot from Bottom Edge to Top Edge
2. Convection from Solid Pot into Liquid
3. Convection from Liquid to Air
4. Convection from Solid Pot to Air
5. Radiation

For the boiling pot example, the ‘overall’ driving force is:

\[ T_{coil} - T_{room} \]

In order to determine how all the heat moves, from the coil into the room air, we can create an equivalent ‘thermal circuit’, as follows:

The ‘thermal circuit’ represents the heat flow in the system, and is analogous to an ‘electric circuit’
Consider the heat transfer through solid bodies (i.e. thermal conduction) in greater detail, with the following diagram:

Here we have:

\[ q_{\text{cond}} = q_{\text{cond}} \cdot \text{Area} = -\frac{\kappa A \Delta T}{L} \]

Using the electrical analogy, we can define thermal resistance as:

\[ R_{\text{th}} = \frac{\Delta T}{q_{\text{cond}}} = \frac{L}{\kappa A} \quad \rho_{\text{th}} = \text{thermal resistivity} = \frac{1}{\kappa} \]

Therefore:

\[ R_{\text{th}} = \rho_{\text{th}} \frac{L}{A} \]

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**Example: Thermal Characteristics of a Beam:**

Question: What is the temperature difference from the heater tip to the substrate, given the current \( i \) through the metal layers?

- Parameters Given:
  - Substrate temp: 21°C
  - Current \( i \) to heater: 20 mA
  - Heater elec. resistance: 10 Ω
  - Beam length \( L \): 300 um
Example: Thermal Characteristics of a Beam:

Beam cross-section A-A:

- Cross-section parameters:
  - $t_b$: 1 um
  - $t_m$: 0.5 um
  - $w_b$: 20 um
  - $w_m$: 8 um

Equivalent ‘heat circuit’:

See Class Notes:
This principle will be used when we need to determine:

(a) How much heat is stored in an object

(b) How quickly the heat will enter or leave the object

The main formula for heat storage is:

$$Q = sh \cdot m \cdot \Delta T$$

where:

- $sh = \text{specific heat} \left( \frac{\text{Joules}}{\text{Kg} \cdot \text{K}} \right)$
- $m = \text{mass}$
- $\Delta T = \text{change in body temperature}$

Common Values for Specific Heat ($sh$) and Density:

<table>
<thead>
<tr>
<th>Substance</th>
<th>Specific Heat (J/Kg*K)</th>
<th>Density (Kg/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air @ 50°C</td>
<td>1046</td>
<td>1.25</td>
</tr>
<tr>
<td>Water @ 0°C --&gt; 100°C</td>
<td>4186</td>
<td>1000</td>
</tr>
<tr>
<td>Aluminum</td>
<td>900</td>
<td>2700</td>
</tr>
<tr>
<td>Gold</td>
<td>130</td>
<td>19,280</td>
</tr>
<tr>
<td>Nickel</td>
<td>444</td>
<td>8910</td>
</tr>
<tr>
<td>Silicon (Bulk)</td>
<td>700</td>
<td>2330</td>
</tr>
<tr>
<td>Polysilicon</td>
<td>753</td>
<td>2330</td>
</tr>
<tr>
<td>Silicon Nitride (SiN)</td>
<td>700</td>
<td>2900</td>
</tr>
<tr>
<td>Silicon Oxide (SiO2)</td>
<td>1000</td>
<td>2200</td>
</tr>
</tbody>
</table>
Define the thermal heat capacity as:

\[ Q = C_{TH} \cdot \Delta T \]

where: \( C_{TH} = sh \cdot m \)

Note that thermal heat capacity is analogous to electrical capacitance.

Thermal Energy Storage

These equations will allow us to determine the amount of time required for heat to flow into an object.

**NOTE: These are dynamic equations, i.e. representing phenomena that change with time.***

Thermal time constant, \( \tau \), is defined as:

\[ \tau = R_h \cdot C_{TH} \]

We can consider the analogous circuit as:

\[ \text{A} = T_{in}, \quad \text{B} = T_{out} \]

\[ R_{in}, \quad C_{in} \]

Thermal Body
Define Temp difference across “thermal capacitor” as:
\[ \Delta T_C = \frac{Q}{C_{TH}} \]

Define Temp difference across “thermal resistance” as:
\[ \Delta T_R = R_{TH} \cdot q \]

Therefore, Temp difference across “thermal body” is:
\[ \Delta T_B = T_H - T_L = \Delta T_C + \Delta T_R \]

Which can be re-written as:
\[ \Delta T_C = -q_C R_{th} + \Delta T_B \]

Only the thermal capacitor can store heat, whereas the thermal resistor impedes the flow of heat.

The flow of heat, \( q \), is defined as:
\[ q = \frac{d}{dt} (Q) \]

The total heat in the thermal capacitor is:
\[ Q_C = C_{TH} \cdot \Delta T_C \]

Therefore, heat flow into the capacitor is:
\[ q_C = C_{TH} \cdot \frac{d}{dt} (\Delta T_C) = C_{TH} \cdot \Delta T_C' \]
Thermal Rise Time

Substituting this into $\Delta T_C = -q_C R_{th} + \Delta T_B$, we will obtain:

$$\Delta T_C = -\Delta T_C' C_{TH} R_{th} + \Delta T_B$$

Rearranging:

$$\Delta T_C' = -\frac{\Delta T_C}{C_{TH} R_{th}} + \frac{\Delta T_B}{C_{TH} R_{th}}$$

This is a differential equation, so solving it at $\Delta T_C(\tau = 0) = 0$ we obtain:

$$\Delta T_C = \Delta T_B \left( 1 - e^{-\frac{t}{C_{TH} R_{th}}} \right)$$

The above is our ‘working equation’ for determining the thermal rise time of an object.

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Thermal Rise Time

This can be plotted as:

$$\Delta T_C = \Delta T_B \left( 1 - e^{-\frac{t}{C_{TH} R_{th}}} \right)$$
Thermal Fall Time

**NOTE: These are dynamic equations***

Thermal time constant, \( \tau \), is defined as:

\[
\tau = R_{th} \cdot C_{TH}
\]

Consider the analogous circuit, which is equivalent to the previous RC system, however, at time \( t=0 \), we ‘short the leads’ as follows:

At \( t=0 \), ‘short leads A and B’

Define: \( \Delta T_c = \frac{Q}{C_{TH}} \), and define: \( \Delta T_r = R_{TH} \cdot q \)

Since we can equate points A and B, we have:

\[
R_{TH} \cdot q = -\frac{Q}{C_{TH}}
\]

Re-arranging:

\[
q = -\frac{Q}{R_{TH} \cdot C_{TH}}
\]

Since we know that \( q = \frac{d}{dt} (Q) \), therefore, we obtain:

\[
q' = -\frac{q}{R_{TH} \cdot C_{TH}}
\]
Thermal Fall Time

Solving the previous differential equation, we obtain the ‘working equation’ for determining the thermal fall time for heat flow:

\[ q = q_o \cdot e^{\frac{-t}{RC}} \]

By a similar derivation, we can derive a differential equation for ‘change in temperature’ of the thermal capacitor (i.e., change in body temperature), as:

\[ \Delta T' = -\frac{\Delta T_C}{R_{TH} \cdot C_{TH}} \]

Which can be solved to obtain the ‘working equation’ for determining the thermal fall time for body temperature:

\[ \Delta T_C = \Delta T_{C_o} \cdot e^{\frac{-t}{RC}} \]

where: \( q_o = initial \ heat \ flow \)
\( \Delta T_{C_o} = initial \ body \ temp \ w.r.t. \ ambient \)

This can be plotted as:

\[ q = q_o \cdot e^{\frac{-t}{RC}} \text{ or } \Delta T_C = \Delta T_{C_o} \cdot e^{\frac{-t}{RC}} \]